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OPTICAL LECTURES

Read in the

PUBLICK SCHOOLS

OF THE

University of *CAMBRIDGE*,

Anno Domini, 1669.

By the late Sir *ISAAC NEWTON*,
Then Lucasian Professor of the Mathematicks.

Never before Printed.

Translated into *English* out of the Original *Latin*.

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M.DCC.XXVIII.

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THE

PREFACE.

I*T was as long ago as the Year* 1666, *when Sir* Isaac Newton *first found out his* Theory of Light and Colours. *Upon Dr.* Barrow's *resigning to him the Professorship of the Mathematicks at* Cambridge, *he made* A. 1669, *this Discovery the Subject of his publick* Lectures, *in that* University. *In* 1671 *he began to communicate it to the World, as also a Description of his* Reflecting Telescope, *in the* Philosophical Transactions. *About the same time he intended to publish his* Optical Lectures, *wherein these Matters were handled more fully; together with a* Treatise of Series and Fluxions. *But the Disputes, which were occasioned, by what he had already suffered to come a-*

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broad, deterred him from that Design. And hence he conceived so great an Horror for any thing, that looked like Controversy, that the constant Importunities of his Friends could not prevail upon him to print his Book of Opticks *until the Year* 1704. *As to his* Lectures, *they were deposited, at the time they were read, amongst the* Archives *of the* University. *From whence many Copies have been taken, and handed about by the Curious in these Matters.*

These Lectures *are divided into two Sets or Parts. What is treated of in the last Part, relates to the* Doctrine of Colours, *and was left imperfect; but has been since published in the* Opticks *by Sir* Isaac *himself with great Improvements. The first Part is compleat and preparatory to the other: And as it contains but little in common, with what has been already printed, we have thought fit to make it now publick. The Reader will find in it Abundance of Particulars worthy their great Author, and such as will even at present appear entirely new. It is divided*

ded into four Sections. What is delivered in the two first Sections may be immediately seen in the Book itself; for there the Author has put in the Margin the Contents against their proper Places. But a short View of the Whole take, as follows. The first Section gives us a very full and plain Account of the different Refrangibility in the Rays of Light, with the Experiments from whence it was deduced; and, amongst many other curious things relating thereto, an elegant Demonstration of the Case, where the Image of the Sun made through a Prism would be circular, provided the common Opinion of Refractions was true. The Subject of the second Section is the Measure of Refractions in transparent Substances, as well Fluid as Solid, and the comparing the Refractions of heterogeneal Rays; and these are performed not only in Mediums as contiguous to the Air, but when contiguous to one another; all which are illustrated by a Description of the Instruments for making the Experiments, and by Exam-

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ples, together with ſuitable Demonſtrations.

The other two Sections are in a manner purely Geometrical. *In the firſt of them the Effects of the Refractions of Rays are conſidered, as they are incident upon one or two plane Surfaces. The firſt nineteen* Propoſitions *relate to the Refractions made by a ſingle Plane. Of theſe the firſt eight treat of homogeneal Rays; containing ſome of the Principles of* Dioptricks. *In the* Scholium *to the eighth* Propoſition *our Author has a curious Speculation concerning the apparent Place of the Image of an Object ſeen by Refraction. The reſt of theſe* Propoſitions *are about the Divarications and Limits of heterogeneal Rays; as they are refracted at a Surface ſeparating two Mediums, whoſe Denſities are conſidered either as permanent, or the Denſity of any one of the Mediums is ſuppoſed to be varied; amongſt which occurs, at the Concluſion of* Prop. XII. *what is very remarkable, that, in Rays of every Sort refract-*

ed at the ſame Point of a plane Surface, the Locus *of the Centers of their Radiations, is the vulgar* Ciſſoid. *From thence to the End of this Section is concerning the Affections of both homogeneal and heterogeneal Rays refracted by two Planes; which chiefly have Relation to the Experiments of the Priſm, from whence our Author deduced his* Theory of Light and Colours. *And here is ſhewn, at* Prop. XX. *and* XXI. *that, if Rays diverge to a Priſm, the homogeneal ones, after the double Refraction, will ſtill continue to diverge, but ſome of the heterogeneal ones will converge; therefore at* Prop. XXII. *that of Rays ſo refracted from an Object to the Eye, ſome will fall gradually nearer to the Vertex of the Priſm than others, as they are more and more refrangible; whence from* Prop. X. *are defined the Orders of Colours in the Image made by Refraction; at* Prop. XXIII. XXIV. *that the bigger the vertical Angle of the Priſm, or the denſer its Matter is, the Difference of the Refraction will be ſo much the greater, whence the Colours*

in the Image will become the more manifest; at Prop. XXV. XXVI. *that the Rays so falling on the Prism, that the Refraction on each Side may be equal, in homogeneal Rays, the Angle, which the incident and emerging Rays comprehend, will then become the greatest; but in heterogeneal Rays the Difference of those Angles will become at that time the least. And at the last* Proposition *our Author sets down a Mechanical Solution of the following* Problem; *Rays being refracted from one given Point to another given Point by a Prism given in Position; to find the Angles comprehended by the heterogeneal Rays. He says, to perform this geometrically, would require such a Construction, as the* Ancients *called* Linear, *or what could not be effected by the Help of the* Conicks.

The last Section regards Rays, as they are refracted by curve Surfaces. Its chief Contents are, at Prop. XXIX. XXX. XXXII. XXXIII. *the finding both the principal* Focus, *and also that of every particular Ray, not only*

only in Spheres, but in any curve Surface whatever; at Prop. XXXI. *the computing the Errors arising from the Figures of Optical Glasses; at* Prop. XXXIV. *the Invention of such Curves, as will accurately refract the Rays of Light to any given* Focus; *and at* Prop. XXXV. XXXVI. *the determining the Rain-bow. In all this Section our Author makes no mention of heterogeneal Rays, until he comes to the last* Proposition, *wherein he determines the Errors caused by the different Refrangibility of the Rays of Light. And from this* Proposition *compared with the thirty first he deduces this Conclusion, that the Imperfection of Optical Instruments is not owing, as has been all along thought, to the Unfitness in the Figure of the Glasses, but to the different Refrangibility in the Rays of Light. This Consideration put our Author upon the noble Invention of the* Reflecting Telescope; *a very particular Account of which is given in his Opticks. This* Instrument *has been lately made to a greater Length, with a very curious*

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Contrivance for managing it by that most ingenious Gentleman John Hadley, *Esq; A Description whereof is published in the* Philosophical Transactions N° 376.

In the Preface of the learned Dr. Barrow's Lectiones Opticæ *printed in* 1669, *there is given a great Character of Sir* Isaac Newton, *at a Time when he was altogether unknown to the World. And the Dr. is so candid as farther to declare, that he there altered several Things upon his Advice, and inserted some of his Inventions, as an additional Ornament to his own. What Dr.* Barrow *published then of our Authors without their Proofs, the Reader will find demonstrated in the following Discourse. But it is needless to cite at this time of Day any Testimonies to our Author's Praise, in order to recommend what shall come from One, who has acquired so universal a Reputation. It will be sufficient, that the present* Treatise *is a faithful Translation of a very correct Copy, taken from the* Latin Original, *as it was read in* 1669.

1669. *We have put it into the Language, Sir* Isaac Newton *himself chose to make Use of in his* Opticks; *and at the Bottom of the Pages we have added here and there some very short Remarks, which, we presume, will not be altogether useless to such, as are not throughly informed of these Matters. By this our Labour we question not, but we shall merit the Thanks of all, who are curious in these Speculations. And we shall moreover proceed in the same manner to deserve well at their Hands; for we hope shortly to present the Publick with several* Mathematical Pieces, *that were long ago written by our great Author, though never yet printed; whereby will be given still farther Proofs, how early that* Genius *exerted itself, which was at length able to produce those divine* Works, *the* Principia *and the* Opticks.

London, June 29.
1727.

OPTICKS

ERRATA.

PAGE 8. Line 20. put *Fig.* 3. in the Margin. p. 86. l. 23. read F *e* and F *c*. p. 88. l. 22. *r.* Refraction, G X will be the Sine of Incidence and K L the Sine of Refraction. p. 94. l. 14. *r.* of. p. 112. l. 19. for E D G *r.* D G E. p. 117. l. 1. *r.* Radiation of the Ray R O. l. 7. *r.* dissimilar. p. 130 l. 9. *dele* the Comma after D K. p. 146. l. ult. *r.* and into. p 147. l. 17. for Density *r.* Thickness. p. 153. l. 7. *r.* Perpendicular I G. p. 155. l. 19. *r.* Refraction. p. 164. l. 4 *r.* F O R. l. 17. *r.* 2 *g* G < p. 165. l. 3. put *Fig.* 51 in the Margin. p 166. l. 15. put *Fig.* 52 in the Margin l. 22. *r.* and farther. p. 175. in the Citation l. 1. *r.* C K : : p. 178. in Note 4. for convex Glasses *r.* Lens's. p. 182. l. 14. put a Point after *a a*. p. 183. l. 7. *r.* I R — I I. l. pen. *r.* N G q. p. 184. l. 19. for *g n.* r. *ng*. p. 185. in the Citation l. 5. *r.* 1665 and 1666. p. 186. l. 7. for I K *r.* G K. l. 8. for R I — I *y.* *r.* R — I. p. 195. l. 9. for Incidence *r.* Refraction. l. 10. for Refraction *r.* Incidence. p. 202. l. 1. *dele* is. p. 203. l. antep. *r.* B C q. p. 204. l. 3. *dele* the Point after N.

N. B. The Numbers in the Computation at Pages 209 and 210 are erroneous, occasioned by a small Mistake made in *Prop.* XXXI. For the Value of P Q is not $\frac{R y^3}{4 I a a}$ as there set down, but $\frac{R R y^3}{4 I I a a}$. This did not proceed from any Fault in our Author's Method of Reasoning, but only from his putting in *Corol.* 6. at Page 186. $\frac{R a}{R - I}$ the Value of C F instead of $\frac{I a}{R - I}$ the Value of B F, from which it differs only by a Letter. This being rectified the Value of P Q will come out, as I have here made it. For in that *Corol.* it will be $\frac{I a}{R - I} . y :: \frac{R R y y}{8 I R a - 8 I I a} \left(= \frac{1}{4} K F\right). \frac{R R y^3}{8 I I a a}$, which is equal to o Q, the double whereof $\frac{R R y^3}{4 I I a a}$. will be the Value of P Q. I made no Alteration of the Numbers in the Manuscript; because the Reader may find a true Computation from more accurate Observations of the Laws of Refractions in our Author's Opticks *Prop.* VII. *Book* I. Part. I.

OPTICKS.

Of the Refractions of the RAYS *of* LIGHT.

SECTION I.

The Refrangibility of Rays is different.

THE late Invention of Telescopes has so exercised most of the Geometers, that they seem to have left nothing unattempted in Opticks, no room for farther Improvements. And besides, [a] since the *Dissertations*, which you have not long since heard from this Place, were composed with so great a Variety of optical

[a] Viz. Dr. *Barrow's Lectiones Opticæ.*

Matters, ſuch Plenty of new Diſcoveries, and that confirmed by moſt accurate Demonſtrations; it may ſeem a vain Endeavour and an uſeleſs Labour, if I ſhall again undertake the handling this Science. But ſince I obſerve the Geometers hitherto miſtaking in a particular Property of Light, that belongs to its Refractions, tacitly founding their Demonſtrations on a certain Phyſical Hypotheſis not well eſtabliſhed; I judge it will not be unacceptable, if I bring the Principles of this Science to a more ſtrict Examination, and ſubjoin, what I have diſcovered in theſe Matters, and found to be true by manifold Experience, to what my reverend Predeceſſor has laſt delivered from this Place.

THEY, that have been converſant in Dioptricks, imagine, that Optical Inſtruments may be brought to any Degree of Perfection, provided they were able to communicate to the Glaſſes in grinding, what Geometrical Figure they pleaſed; and to that Purpoſe various mechanical Contrivances have been thought of, whereby Glaſſes might be ground in-

into hyperbolical or even parabolical Figures; but no Body has yet ſucceeded in the exact Deſcription of ſuch Figures: The Labour has been in vain; and that they may no longer take Pains in a fruitleſs Inquiry, I durſt promiſe them, that although every thing ſhould ſucceed happily; it would, nevertheleſs, not at all anſwer their Expectations. For Glaſſes, tho' they were formed into Figures, the beſt that could be deviſed for that End, yet they would not perform twice as well as ſpherical ones, ground with the ſame Exactneſs. I do not ſay this, as if I contended, that the Writers in Opticks were to blame: For all they have advanced is accurate, and moſt true with regard to the Intention of their Demonſtrations; but however they have left ſomething, and that of the greateſt Moment to be diſcovered by their Succeſſors; as in Refractions I find a certain Irregularity, that diſturbs all things; and not only cauſes, that the Figures of the conick Sections do not much ſurpaſs ſpherical ones, but alſo that ſpherical ones perform much leſs, than they would, if the ſaid Refraction was uniform.

I shall therefore treat of Dioptricks, not that I intend to handle that Subject entirely, but only that I may first search out this Property in the Nature of Light, then shew how much the Perfection of Dioptricks is impeded by this Property, and by what Means that Inconvenience, as far as the Nature of the thing will permit, may be avoided. Where I shall produce something, that regards as well the [a] Theory, as the Praxis both of Telescopes and Microscopes: [b] Shewing that the greatest Perfection of Opticks is to be sought, contrary to the common Opinion, from Dioptricks and Catoptricks joined together. [c] And in the mean while I shall largely explain the Difference of Colours

[a] This is not deliver'd in any of our Author's *Lectures*; but for which see *Huygen's Dioptricks*, printed first in 4[to] at *Leyden*, A. 1703. amongst his *Opuscula Posthuma*.

[b] In our Author's reflecting Telescope, first published in the Philosophical Transactions N° 81. and afterwards in his *Opticks*.

[c] What is here promised is most wonderfully performed in his *Opticks*.

and their Geneſis both from Priſms, and alſo from coloured Bodies.

CONCERNING Light I have diſcovered, that its Rays in reſpect to the Quantity of Refraction differ from one another. Of thoſe, that have all the ſame Angle of Incidence, ſome will have their Angle of Refraction ſomewhat greater, others will have it ſomewhat leſs. For a farther Illuſtration of this, let E F G (in *Fig.* 1.) be any refracting Surface, ſuppoſe of Glaſs, and let any Line O F be drawn meeting it in F, and making with it the Angle O F E acute; conceive alſo the ſolar Rays to flow through that Line O F, continually ſucceſſive to one another, ſo that they may one after another impinge on the Point F, and there be refracted into a denſer Medium; or, if you had rather, ſuppoſe parallel Rays to be indefinitely little diſtant from O F, and to fall on the Points the neareſt to F. Now from the commonly received Opinion, thoſe Rays having the ſame Incidence ought alſo to have all the ſame Refraction; as ſuppoſe into the Line F R. But I have

2. All Rays have not the ſame Refrangibility.

Fig. 1.

diſcovered the contrary, *viz.* that after they are refracted, they diverge from one another, as if ſome were refracted into the Line F P, others into the Line F Q and others into the Lines F R, F S and F T, and innumerable others alſo through the intermediate Spaces; and ſome wandering on one Side and on the other, according as any Ray is ſuited to ſuffer a greater or leſs Refraction. I moreover find, that the Rays F P refracted the moſt produce purple Colours, and thoſe F T the leaſt refracted produce red Colours; but theſe F Q, F R, F S, that proceed intermediate to thoſe, generate the intermediate Colours, *viz.* blue, green and yellow; and ſo the Rays, as they are ſuited, that ſome may be more and more refracted than others, do generate theſe Colours in order, red, yellow, green, blue, and purple, together with all the intermediate ones, that may be ſeen in the Rain-bow; whence will eaſily appear the Production of Colours in a Priſm and the Rain-bow. But theſe things being ſlightly obſerved, I ſhall defer to hereafter, what I have to ſay of Colours.

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OUR Opinion in this Matter being thus briefly explained, that you may not think we have declared to you Fables instead of Truth, we ſhall immediately produce the Reaſons and Experiments on which theſe things are founded: And becauſe a certain very obvious Experiment of the Priſm gave me firſt an Occaſion of diſcovering the reſt, I ſhall firſt explain that. Let (in *Fig.* 2.) F be any Hole in the Wall or Shut of a Chamber Window, through which the ſolar Rays O F may be tranſmitted, all other Avenues being every where carefully cloſed, leſt the Light may enter elſewhere; this darkening of the Room is not neceſſary, but only cauſes, that the Experiment may be ſomething more evident. Then let be applied to that Hole the triangular glaſs Priſm A *a* B *b* C *c*, that may refract the Rays O F tranſmitted through it, towards P Y T Z, where you will ſee formed a very oblong Image, whoſe Length P T is four times its Breadth Y Z, and above. And hence it ſeems certainly to be evinced, that of the equally incident Rays, ſome

3. Proved from a vulgar Experiment by the Length of the refracted ſolar Image.

Fig. 2.

ſuffer a greater Refraction than others; for if the contrary was true, the ſaid Image of the Sun would appear nearly circular, and in a certain Poſition of the Priſm would be ſeen altogether as to Senſe circular, which is repugnant to all Experience; for in whatever Situation I diſpoſed the Priſm, I could never yet bring about, but that the Length of the Image was more than Quadruple its Breadth, the Angle of the Priſm A C B or *a c b* being about 60 Degrees.

4. A Caſe wherein the equally refrangible Rays make a circular Image.

BUT that there is a certain Poſition of the Priſm, in which the Image of the Sun according to the received Opinion about Refractions would appear circular, I thus ſhew. Near the Hole made in the Window-ſhut, let be placed abroad a Priſm, or what comes to the ſame thing, let E G be an opake Body placed on this Side the Priſm, in which let F be an indifferently ſmall and round Hole, through which the refracted Rays may be tranſmitted to the directly oppoſite Wall to paint there the Image P Y T Z, and let A B C be ſuppoſed to be a Plane cutting the refracting Planes

a C

a C and *b* C perpendicularly, and also passing through the Hole F, as likewise through the Center of the Sun D I H V, which let it bisect according to its Diameter D H, from whose Extremities let proceed the Rays D K and H N, lying in the same Plane, which after they are refracted, D K into K *n* and *n* T, and H N into N *k* and *k* P, they both may go forward through the Center of the Hole F; and besides, let there be such an Inclination of the Prism to those Rays, that the Angles A K D and B *k* F may become equal. Then let I V be another Diameter of the Sun perpendicular to the said Plane A B C, from whose Extremities let two other Rays V L and I M proceed, one I M on this Side the Plane A B C, which may be refracted into M *l* and *l* Y, but the other V L, beyond that Plane, which may be refracted into L *m* and *m* Z, and let all the said four Rays intersect one another in the middle of the Hole F; lastly let it be supposed, that the lucid Image P Y T Z does directly respect the Hole, so that F P and F T, also F Y and F Z may be equal. I now say, that

that in ſuch a Poſition of the Priſm the Angles P F T and Y F Z are equal, on a Suppoſition that all the Rays are equally refracted, that have the ſame Angle of Incidence, and therefore that this Image at leaſt to Senſe ought to be circular, as whoſe Diameters P T and Y Z interſect one another perpendicularly, and ſubtend thoſe equal Angles.

5. A Demonſtration of this Caſe, its 1ſt Part. BUT that thoſe Angles PFT and Y F Z are equal, I thus demonſtrate. Conceive any Ray to go backwards from P through *k* and N, whilſt another Ray proceeds from D through K and *n*; therefore ſince the Angles A K D and B *k* F are ſuppoſed equal, the Angles A K *n* and B *k* N made by the firſt Refractions, will be alſo equal, whence the Triangles C K *n* and C *k* N, will be ſimilar, and their external Angles *k* N A, K *n* B equal, and conſequently the Angles made by the ſecond Refractions A N H and B *n* F are equal. Wherefore ſince the Angles A K D and B *k* F, alſo A N H and B *n* F are equal, their Differences will be alſo equal, that is, the Angle *n* F *k* or PF T equal to the Angle, which the

the Rays D K and H N comprehend, or to the Sun's apparent Diameter. The Angle P F T therefore is equal to the Sun's Diameter. Wherefore ſince it ſhall be moreover demonſtrated, that the Angle Y F Z is equal to the ſaid Diameter, the Propoſition will be manifeſt. But in order to do that, a certain *Theorem* muſt be ſet down by way of *Lemma*.

LET there be (in Fig. 4.) two Planes A B C D and E F G H perpendicular to one another, whoſe common Interſection let be K L, and let I P be any Ray, which falling on the Plane A B C D at the Point P is refracted by it into P R; I ſay, that the Sine of the Angle, which the incident Ray I P makes with the perpendicular Plane F H, is to the Sine of the Angle, which the refracted Ray P R makes with the ſame Plane, as the Sine of Incidence to the Sine of Refraction, and conſequently in a given Ratio. For the Rays I P and P R being taken equal, and I Q and R V being let fall perpendicular to the Plane F H; and moreover at the Point of Incidence the — 6. A *Lemma* to the 2d Part. Fig. 4.

Line

Line SPT being drawn perpendicular to the refracting Plane B D, (which therefore coincides with the other Plane F H) and I S and R T being let fall perpendicular to S P T: The Angle I P Q will be that, which the incident Ray I P makes with the perpendicular Plane F H, and R P V the Angle, which the refracted Ray P R makes with the same Plane. Also I P S is the Angle of Incidence, and R P T the Angle of Refraction. Wherefore if I P or P R be supposed the Radius of a Circle, I Q, R V, I S and R T will be the Sines of the said Angles. But I Q and R V are parallel [a], by reason they are perpendicular to the same Plane F H. Also I S and R T are parallel [b], because lying in the Plane I S P T R they insist perpendicularly on the same right Line S T. That is, the right Lines I Q, I S, which comprehend the Angle Q I S, are parallel to the right Lines R V, R T, comprehending the Angle V R T. Wherefore those Angles Q I S and V R T are equal [c]. But Q S and V T being drawn, the Angles I Q S, and R V T will become right ones; [d] because the right Lines I Q and R V insist

[a] 6. 11. Elem.

[b] 28. 1. Elem.

[c] 10. 11 Elem.

[d] Def. 3. 11. Elem.

insist perpendicularly on the Plane F H. Therefore the Triangles I Q S and R V T are similar,[e] and I Q. R V : : I S. R T, that is, the Sines of the Angles, which the incident and refracted Rays make with any Plane F H perpendicular to the refracting Plane B D, are as the Sines of Incidence and Refraction, and consequently in a given Ratio. For that the Ratio of these Sines is given, * *Cartes* has taught, and others have since experienced. [e] 4. 6. Elem.

MOREOVER the Truth of the Theorem now demonstrated will remain good, though the Plane E G insists perpendicular to the refracting Plane B D elsewhere, than at the refracting Point P. For from thence neither the Angles made with the Rays and Plane F H; nor therefore the Sines of those Angles will be altered.

* At this Time I suppose our Author thought *Cartes* to be the Discoverer of this Proportion, which he deduced from a Manuscript of *Snellius*, as was afterwards hinted by Sr. *Isaac* himself in the *Scholium* to Prop. 96. Lib. 1. of the *Principia*, and also by *Huygens* in his *Dioptricks*.

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7. The second Part. THESE things being thus ſhewn, I now return to what was propoſed, *viz.* to demonſtrate the Angle Y F Z (in *Fig.* 3.) to be equal to the apparent Diameter of the Sun, and conſequently to the Angle P F T. From what was ſaid above it appears, that the Plane K D H N *k* F *n* biſects the Angle contained by the Rays I M and V L lying on each Side. Therefore ſeeing that Angle is equal to the Diameter of the Sun, the Angle, which one of the Rays, ſuppoſe I M, makes with the ſaid Plane, will be equal to the Sun's Semidiameter, whoſe Sine let be A, and B the Sine of the Angle, which that Ray refracted M *l* makes with the ſame Plane. Now ſince that Plane is ſuppoſed perpendicular to the refracting Plane A C of the Priſm, it will be from the foregoing Lemma, as the Sine A to the Sine B, ſo the Sine of Incidence to the Sine of Refraction out of a rarer Medium into a denſer Medium. Or on the contrary, as the Sine of Incidence to the Sine of Refraction from a denſer to a rarer Medium, ſo will be B to A. Wherefore ſince

Fig. 3.

ſince the ſaid Plane D H F is alſo perpendicular to the other Plane B C of the Priſm, which refracts the Rays from a denſer into a rarer Medium, and moreover ſince B is ſuppoſed the Sine of the Angle, which the incident Ray M *l* makes with that perpendicular Plane D H F, A will be (by the preceding Lemma) the Sine of the Angle, which the refracted Ray *l* F makes with the ſame Plane D H F; but A is ſuppoſed the Sine of the Sun's Semidiameter; therefore that Angle, which the Ray *l* F makes with the Plane D H F, is equal to the Sun's Semidiameter, and its double *l* F *m* or Y F Z equal to the Sun's Diameter: And ſince it was ſhewn above, that the Angle P F T is equal to the ſame Diameter, thoſe two Angles Y F Z and P F T will be equal. *Q. E. D.*

Now if the Plane Y F Z was perpendicular to the Plane of the Image P Y T Z as well as the Plane P F T, thoſe four Lines F P, F T, F Y and F Z, which comprehend equal Angles, would be all equal amongſt themſelves, and

and conſequently the Subtenſes P T and Y Z would be alſo equal. But who carefully conſiders this Thing, will find the collateral Rays V L *m* F Z and I M *l* F Y to be a little leſs refracted than the other two D K *n* F T and H N *k* F P, and therefore the Plane Y F Z will decline a little more from the Ray F P than from F T, cutting the Line P T below its middle Point X, and ſo divaricating from the Perpendicular F X (which conceive to be drawn) will be ſomewhat oblique to the Plane of the Image P Y T Z, and for that Cauſe the Lines F Y and F Z will be a little greater than F P and F T, and the Subtenſe Y Z a little greater than the Subtenſe P T. But I omit the Demonſtration of this as being prolix, and not altogether neceſſary to my Purpoſe. For it matters not much, whether the Plane Y F Z is right to the Plane of the Image P Y T Z, or ſomething oblique, that is, whether Y Z is equal to, or greater than P T; it is ſufficient, that it cannot be leſs. Yea, ſince by Reaſon of the *Iſoceles* Triangles P F T and Y F Z it is F P. F Y: : P T.

P T. Y Z, and F P and F Y are very nearly equal, the Difference will be very ſmall between P T and Y Z, that as to Senſe they may be looked upon as equal.

THERE is therefore ſhewn a Caſe, wherein the Length of the Solar Image, tranſmitted through the Priſm, would be beheld equal to its Breadth; and conſequently wherein that Image would appear as circular, provided the vulgar Opinion was true. Moreover, although the Poſition of the Priſm ſhould be otherwiſe than I have deſcribed, ſuppoſing the Rays on each Side did not ſuffer a very unequal Refraction; yet the Figure of the Image would ſcarce on that Account be altered. Nor does it much ſignify, whether the opake Body E G, in which is bored the Hole F to tranſmit the Rays, be placed on this Side the Priſm or beyond it; nor is the Figure of the Hole to be much regarded, ſo it be but ſmall. For ſuch minute Variations will not alter the Image more, than perhaps a tenth or fifth Part of its Diameter, as will appear to

8. But in that Caſe the length of the Image is more than quadruple its Breadth. Whence the different Refrangibility is evinced.

one that considers. And so, that I may comprehend all in a few Words, it is manifest, that the refracted Image of the Sun ought for the most Part to be to Sense, as if it were circular, provided the Refraction of the same Incidence on the same Medium were to be always the same. But the first is repugnant to Experience; its Length exceeding, as it has been said, its Breadth by more than four Times. Therefore the last is repugnant to Truth, and the Refraction of the same Incidence is different.

9. A shorter Demonstration of the same Thing.

FROM the same Experiment I could here thus more briefly have shewn the Proposition, *viz.* since I so disposed the Prism, that the Refraction as well of the imerging as the emerging Rays, might be as it were equal, I measured (*Fig.* 2. or 3.) the Angles P F T, and Y F Z, and found the Angle Y F Z, indeed, equal to half a Degree or the Diameter of the Sun; but the Angle P F T exceeded the same Diameter four times, and above, to which however it ought to be equal, from the former Part

I

H

Plate 1st

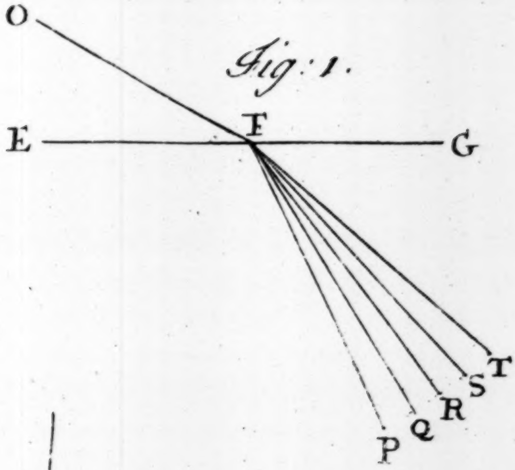


Fig:1.

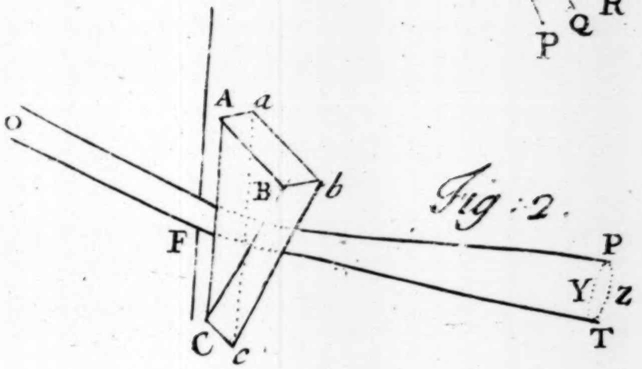


Fig:2.

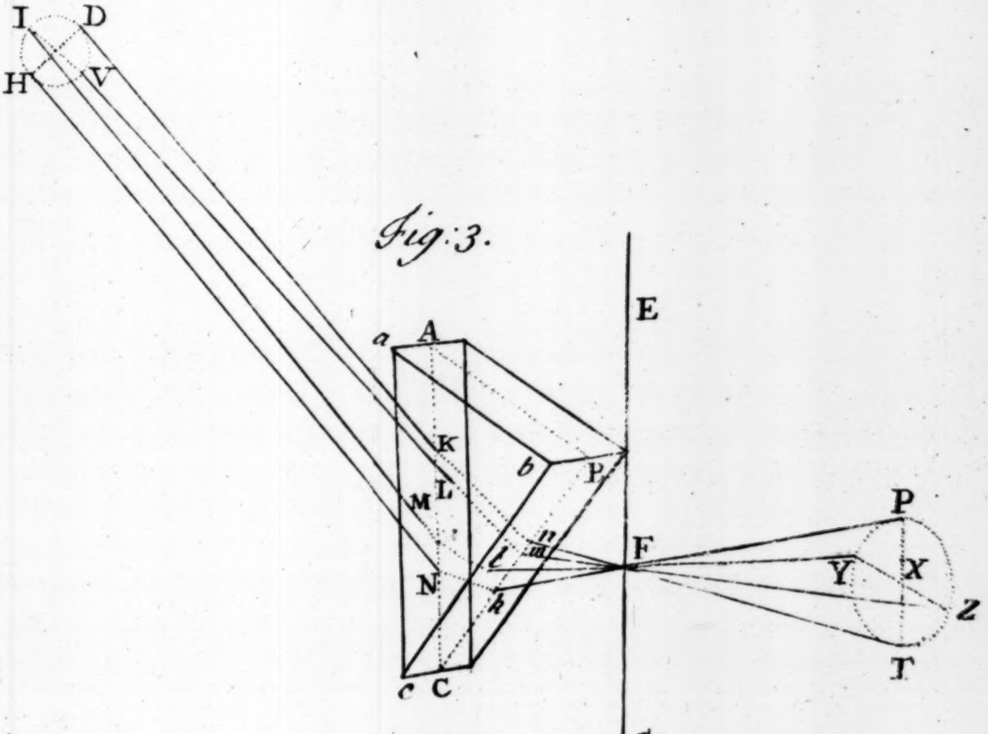


Fig:3.

of the foregoing Demonſtration ; and thence the Propoſition moſt manifeſtly appears. But for the Sake of what will next follow, it ought to be demonſtrated, that thoſe Rays, whoſe Refrangibility is not unlike, will form an Image nearly circular, and on that Account I have thought fit to deliver that Demonſtration, although it be ſomewhat long, for the Illuſtration of this Experiment.

10. By what Means the Priſm may be eaſily placed in a Situation fit for the ſaid Experiment.

BUT ſince in making the ſaid Experiments, I have ſuppoſed the Poſition of the Priſm to be ſuch, as that the Rays may be equally refracted on both Sides of the Priſm: I ſhall, for a Concluſion, declare by what Means, that may be ſoon and eaſily done. If the Priſm be held in the Sun's Light, and by a gentle Motion turned about its Axis, you will ſee the Colours, which it makes, to be by a continual Motion tranſlated from Place to Place, in ſuch a manner, as ſometimes they will appear to aſcend, and then again deſcend. Obſerve therefore the Middle between theſe contrary Motions, when the Colours now aſ-

cending, and presently being about to descend, seem to be stationary, which when you see, stop the Prism, and fix it in that Station. I say it is done, for then the Sum of the Refractions made on both Sides, or the Inclination of the emerging Ray to the incident one, will become the least of all. Which when it happens, the Refractions on both Sides are equal, as shall be demonstrated hereafter. [a]

11. The Figure of the said Image is described, which is comprehended partly by right Lines, partly by Semicircles.

IT is my Design now to prosecute the various Circumstances of this Experiment, not less pleasant to the Experimenter, than declarative of our Purpose. And it is in the first Place to be remarked, that the Figure of this Image, according to its Length, was terminated by straight Lines, and according to its Breadth by two (as well as I could judge by my Sight) Semicircles. In the 5th Figure let P T be the Image of the Sun refracted by the Prism; I observed this to be terminated on its Sides by two Lines A B and C D, straight and parallel to one another, as to Sense, but at the Extremities by two Semicircles

[a] In SECT. III. Prop. XXV.

cles A P C, and B T D, the Cauſe whereof is thus determined, from what has been already declared.

LET thoſe terminating Semicircles be compleated into Circles, as you ſee in the 6th Figure, and let another Circle Y Z be inſcribed intermediate to theſe. Now conceive certain Rays coming from the Sun, that are ſo diſpoſed as being equally incident, they are alſo equally refracted. Theſe being tranſmitted through the Priſm, from what has been demonſtrated above will delineate (if they could be ſeen alone) as to Senſe a circular Image, as B D. Then conceive other Rays of the ſame Sun alſo uniform to one another, that are ſo diſpoſed as to be a little more refracted than the former. Theſe therefore will paint another circular Image, as Y Z: And conceive other Rays ſtill more refrangible, that ſhall produce a third Image A C. Laſtly, imagine innumerable other Rays more and leſs refrangible, than the foregoing, and they will there form alſo other innumerable circular Images both interme-

12. How this ariſes from the circular Images (which every Sort of equally refrangible Rays produces) diſpoſed length Ways. *Fig.* 6.

diate and extream to the former ones, illuminating the oblong Space P Y T Z, contained by two right Lines A B and C D, and two Semicircles. But ſince thoſe Images are all nearly of the ſame Magnitude, and diſpoſed in a right Line between the Lines A B and C D, theſe Lines A B and C D, may be looked upon as right Lines parallel to one another, and as to Senſe they will appear ſuch; and ſo the whole Space P Y T Z illuminated by Rays variouſly refracted from the ſame Incidence will be terminated partly by parallel ſtraight Lines, and partly by oppoſite Semicircles, as is found by Experience.

13. Hence is deduced an Experiment, whereby the ſtraight Borders become very diſtinct.

BUT that I might fully confirm this Conjecture, I bethought myſelf of the Image of the Sun tranſmitted without any Refraction through a Hole to a great Diſtance, as that it is ill defined, its Termination between the Light and Shade being very indiſtinct: But if thoſe Rays paſs through a convex Lens, whoſe Focus is at the Image, the Image will be terminated moſt diſtinctly. After the ſame Manner I perceived of equally refran-

frangible Rays, that if they were transſmitted through a Priſm to a great Diſtance, they would paint a circular Image ill defined, whoſe Termination notwithſtanding by the interpoſition of a convex Lens would become moſt diſtinct. Therefore when I ſaw the Terminations of the refracted Image P Y T Z not very diſtinct; of the Images B D, Y Z, A C, and the reſt forming that oblong one I conjectured, that being tranſmitted through a convex Lens they would be terminated much more diſtinctly than otherwiſe; and upon Trial it proved ſo. For the right Lines A B and C D, in which all thoſe circular Images on each Side were terminated, I ſaw very diſtinct, which before I had ſeen confuſed.

BUT what ſeems very remarkable, the circular Terminations A P C and B D T of that Image always appeared very confuſed, the Light decreaſing by Degrees, till at length it ended in the Shade; *viz.* the intermediate Circles as Y Z, are mixed with other Circles falling on each Side, with which they 14. Why the circular Terminations always appear confuſed.

coincide in ſome of their Parts; but the circular Extremities A C and B D concur indeed with the others but in one Part, and their Meeting becomes continually rarer, and from thence the Light fainter, until it arrives at he Extremities P and T. But there is another Cauſe of this, which is, that the greateſt plenty of Rays is ſuited to ſuffer a mean Refraction, and ſo to fall in the middle of the Image, and that their Number becomes continually leſs, to which belongs on each Side a more extream Degree of Refraction.

15. An Admonition concerning the Figure and Situation of the Lens and Priſms.

BUT in order to try theſe things I adviſe Lens's to be uſed, whoſe Foci are remote, being diſtant perhaps ſix or twelve Feet from the Lens's, provided ſuch are at hand; at leaſt let them not be leſs diſtant than two Feet. And alſo the Sides of the Priſm ought to be exactly plain; but if its Sides are ſomewhat convex, then it is more eligible to make uſe of a Lens, whoſe Focus is diſtant from it two or three Feet only. Theſe things being provided, place the Lens near the Priſm on either

either Side in ſuch a Manner, that it may directly reſpect the Rays tranſmitted through it. Then let the Rays be received upon a Paper, which move backwards and forwards till you ſee the coloured Image terminated on both Sides moſt diſtinctly by parallel ſtraight Lines. But it muſt be obſerved, that when the Priſm is placed beyond the Hole F, as in *Fig.* 3. or on this Side very near it, and the Lens is farther diſtant from that Hole than the Focus of the Lens, which would be made by the Rays falling parallel on it, is diſtant from the ſaid Lens; you will find two Caſes, in which the Image projected on the Paper will become diſtinct; one, when all the homogeneal Rays, that fall parallel upon the Lens, are ſo refracted, as that they concur on that Paper in the ſame Point; which happens, when you ſee the Image coloured, oblong, and terminated diſtinctly by parallel ſtraight Lines. The other Caſe is, when all the homogeneal Rays diverging from one Point of the Hole F, after they are refracted by the Lens, do converge again to one Point of the ſaid Paper. But

16. And of a certain circular Image.

But this happens, when you ſee the Image white, circular, and well defined on all Sides, concerning which I ſhall ſpeak more fully elſewhere. It is ſufficient to have given this Admonition in this Place, that any one experiencing this with his own Eyes, may not be uncautiouſly deceived by the Ambiguity of the Effect, and thence call in Queſtion, what has been ſaid above.

17. And of the Shadows of Clouds interpoſing between the Sun.

I ſhall further obſerve, that ſome very thin Clouds have intercepted the Sun's Diſc, not quite obſcuring the ſame, and have caſt Shadows on this Image P T, not like to themſelves, but extended in Length and parallel to the rectilineal Edges of the Image, which agrees accurately with the Reaſons juſt now alledged. For (*Fig*. 7.) conceive ſome Cloud on the Sun's Diſc to appear like the Sun itſelf, and let it, if the Rays moſt refrangible and bounded by the Circle A C are beheld, project a Shadow into the Place L; ſo that the Circle A C with the Shadow L may expreſs the Sun's Diſc obſcured by a Cloud. Which being ſuppoſed, if the

Fig. 7.

the Rays the least refrangible and circumscribed by the Circle B D, be beheld, the Shadow of the Cloud will be cast by them into the Place N, whose Situation in the Circle B D will be such, as is that of L in the Circle A C, *viz.* this also represents the Sun's Disc obscured by a Cloud. And the same is to be likewise understood of any intermediate Circle with its small Shadow M. So that on Account of the indefinite multitude of Circles possessing the whole Space A B D C, the Cloud may disperse its Shadows through the whole Length L N, and render it obscure; and so, since many Clouds or Parts of Clouds may intervene between the Sun, its Image will be obscured by many Clouds diffused in Length and Parallel.

THAT I might search out, as diligently as possible, the said Proprieties of Light, I farther devised the following Method, by which I might bring them to an Examination, *Viz.* (in *Fig.* 6.) since the Magnitude of the Circles A C, Y Z, B D depends on the Magnitude of the Sun, if the Sun's Diameter could

18. From the Figure of the Image another Experiment is also deduced, whereby it becomes much more oblong. *Fig.* 6.

could be made ſomewhat leſs than it now really is, then thoſe Circles alſo would become leſs, without the Diſtance of the Centers H, I, K, being altered at all, as may be ſeen in *Fig.* 8; and ſo the Breadth of the Image compared with its Length, would become much leſs than before, as both being diminiſhed by the ſame Quantity. To prove this, I cauſed the Rays of the Sun to paſs through two ſmall Holes very diſtant from one another, before they fell upon the Priſm; by which means the Rays coming from the extream Parts of the Sun were excluded, and the Affair ſucceeded, as if the Sun's Diameter had been really diminiſhed. For Illuſtration, let in *Fig.* 9. *e f g* be the Window-ſhut perforated with a ſmall Hole *f*, by which the ſolar Rays may enter the Chamber otherwiſe darkned; then let E F G be an opake Body perforated at F, and ſo placed in the middle of the Room, that the Rays may again paſs through that Hole, before they arrive at the Priſm A B C placed behind. Now the Diameter of theſe Holes being $\frac{1}{8}$ of an Inch, and their

Fig. 8.

Fig. 9.

their Diſtance *f* F, being 12 Feet (*viz.* that the greateſt Inclination of the Rays paſſing through both the Holes might be an Angle of nearly 6 Minutes, that is, about the fifth Part of the Sun's Diameter) and alſo the Image P T projected upon a Paper 10 Feet diſtant from the Priſm, as the the Smallneſs of the Room would permit; I found the Length of the Image to be more than four Inches and an half, and the Breadth the third Part of an Inch; that is, the Length more than 14 times greater than the Breadth, as it ought to be, from what has been ſaid. For ſince thoſe Rays only are let in, that are inclined to one another leſs than the fifth Part of the Sun's Diameter, the Diameters A C, Y Z and B D, diminiſhed by the Diameter of the Hole F, ought to be five times leſs, than otherwiſe they would be, as is to be ſeen in *Fig.* 6 and 7. As if *Fig.* 6, 7. they were produced by a Sun, whoſe Diameter was five times leſs than that of our Sun. But if the opake Body *f g* (*Fig.* 9.) was removed, that the *Fig.* 9. Rays might paſs only through one Hole

F to

F to the Priſm, as was done in the former Experiments, the Breadth of the Image would become $1\frac{1}{6}$ of an Inch; and the Length more than 5 Inches; the Angle of the Priſm being 60 Degrees or a little more. Therefore the Diameter of the Circles A C, Y Z and B D, which conſtitute the Image after the Manner declared above, would be $1\frac{1}{6}$ Inch; from whence let be taken the Diameter of the Hole, *viz.* $\frac{1}{8}$ Inch, and there will remain $1\frac{1}{24}$ Inch, to whoſe fifth Part let there be again added the ſame Diameter of the Hole or $\frac{1}{8}$ Inch, and there will be produced $\frac{1}{3}$ Inch, the Diameter of the Circles A C, Y Z and B D in Figure 8; which is leſs than the Diameter of
Fig. 8. thoſe Circles in *Fig.* 6, by the Quantity
Fig. 6. $\frac{1}{6}$ of an Inch; wherefore the 7 Figure
Fig. 6, 7. is every where leſs than the 6, by the Quantity $\frac{1}{6}$ Inch; and therefore its Length becomes more than 4 Inches; but its Breadth the third of an Inch. The ſame might alſo in ſome Meaſure happen, from the Image P T being farther removed from the Priſm. But it is to be obſerved, that I ſuppoſe the Holes

f and

f and F respecting the Rays directly, although it does not much import, whether their Situation be a little oblique, as is done in the ninth Figure annexed.

MOREOVER, if in this Experiment you shall make Use of, as before, a convex Lens, whose Focus falls at the Image, the Hole F, if you please, being enlarged, or the opake Body E S entirely removed, that the Rays may only pass through the distant Hole *f*, and if you shall make the Hole *f* narrower than before, the rest remaining the same as at first, you will see a very long Image, and for its Length more lucid, than in the former Case. For example, if the Diameter of the Focus be the twentieth Part of an Inch, and if you shall place the Prism with the Lens twelve Feet distant from thence, you will see the Length of the Image more than eighty or an hundred Times greater than the Breadth. But in making these Experiments the Room ought to be every where closed, lest the Light entering at any other Place be- 19. This Experiment is promoted.

besides the Hole *f* may disturb the Image, and render it obscure near its circular Extremities. And moreover if the Surfaces of the Prism are accurately plane, a Lens ought to be made use of, that projects its Focus to a great Distance, suppose to 12 or 20 Feet, provided the Bigness of the Room will permit, by which means you may make a more certain Judgment of the Proportions of the Image. But if the Sides of the Prism are somewhat convex, as it sometimes happens in those, that are commonly sold; such a Prism may be used alone without any Lens, and its Convexity will, instead of a Lens, collect the Rays at a great Distance. Besides, if with the Prism you make Use of a small Lens, whose Focus is not distant above two or three Feet, you will see an Image sufficiently long indeed, but whose Breadth will not be sensible. Which does not the less answer our Purpose, as well as if you could accurately judge of the Proportion of the Length to its Breadth. In making also these Experiments it may be farther remarked, that the

Fig 4.

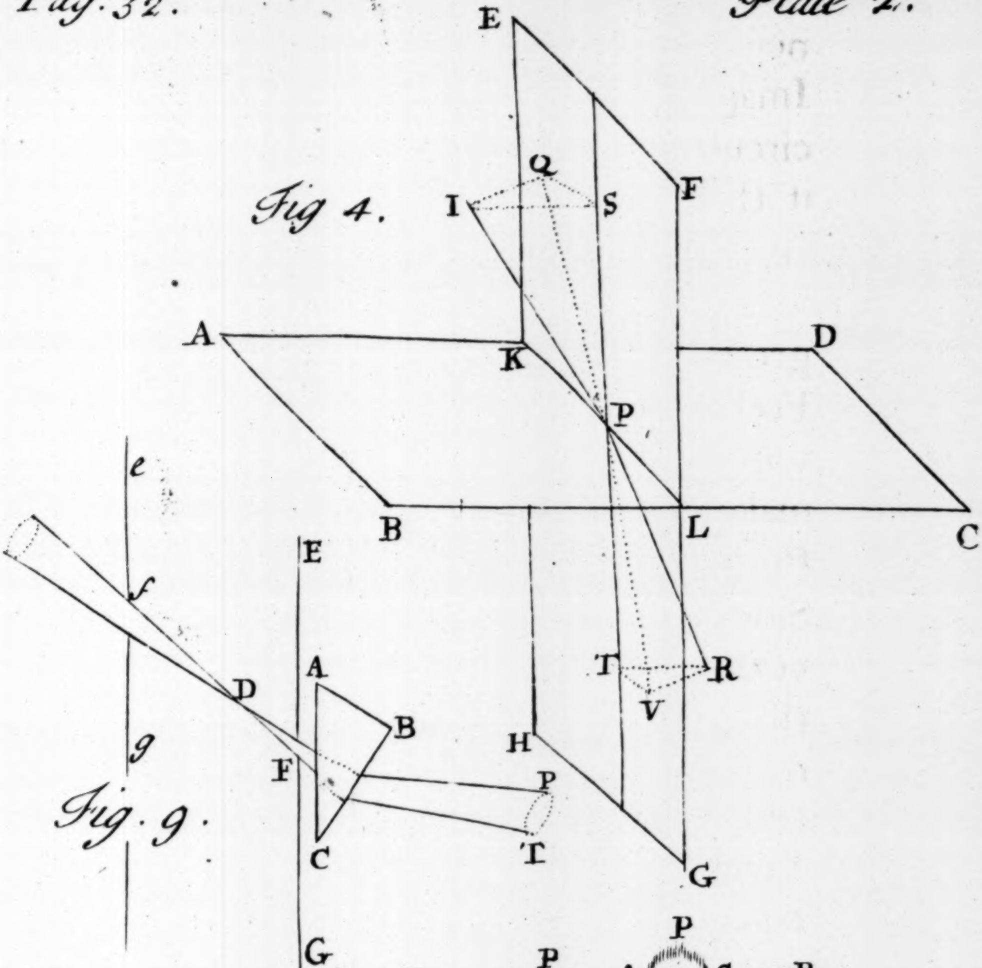


Fig. 9.

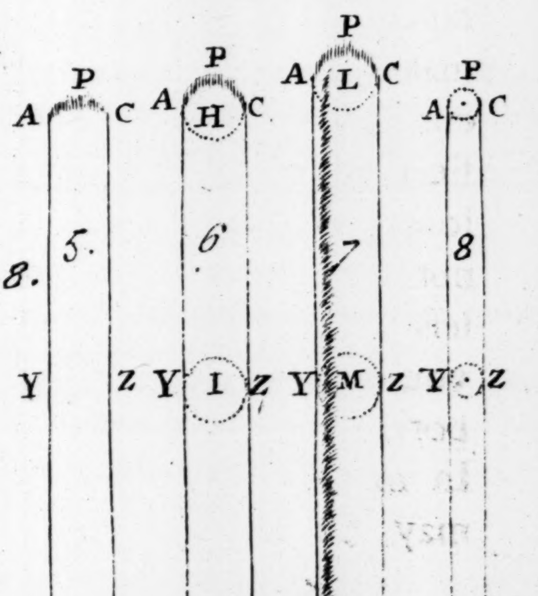


Fig.s 5.6.7.8.

Lens ought not to be placed ſo far behind the Priſm, but that it may be extended to tranſmit all the Rays together, leſt you be obliged to obſerve the Image ſucceſſively by its Parts only. And laſtly it may be obſerved, that if you place the Hole F beyond the Priſm, and the Lens beyond that Hole, at a greater Diſtance from it, than the Focus of its Rays flowing from the more remote Hole *f* is diſtant from the Lens, there will be two Caſes in which the Image caſt upon the Paper will appear diſtinct, according as the Rays coming from every Point of the Hole F, or from every Point of the Hole *f*, are collected in as many Points of the Paper. In one Caſe the Image will be white and circular, as I[a] obſerved before; but in the other Caſe oblong and coloured, as the preſent Experiment requires.

[a] No. 16.

20. This ſtill farther promoted by the Image of the Planet *Venus*.

IT now appears from what has been ſaid, that the Breadth of the Image P T always becomes ſo much the leſs, as the remote Hole *f* is made the narrower, ſo that it is not to be doubted, but

the ſaid Breadth would altogether vaniſh, if inſtead of that lucid Hole, there was one moſt bright Point in its place; and that this would be ſo is confirmed from a not unlike Obſervation, which I formerly made of the Planet *Venus*. The room being every where cloſed, except a Hole a little more than two Inches broad, that it might be made very dark: In this Hole I placed the Object-Glaſs of a ſeven Foot Teleſcope, its Aperture being two Inches and more broad to tranſmit a ſufficient Quantity of Rays. Then at the Diſtance of ſeven Feet a Paper being placed tranſverſely, I ſaw projected upon it the Image of the Planet like a lucid Point; and a Priſm being interpoſed at the Diſtance of one or two Feet from that Paper, by which the tranſmitted Rays might be refracted again: Inſtead of that lucid Point, I ſaw at the Diſtance from thence of more than a Foot a very fine Line, tho' not very bright, however very eaſy to be diſcerned, and whoſe Length exceeded half an Inch, but its Breadth as to Senſe, was none; at leaſt not greater, than juſt to be per-

ceiv'd. And I believe the ſame thing might be obſerved of Stars of the firſt Magnitude, as of *Sirius*, eſpecially if a Lens be uſed four or ſix Inches broad, that it may tranſmit many Rays.

How well this Experiment agrees with our Explanation, which we gave at the Beginning, of the different Refraction of Rays incident with the ſame Angle, it may be worth while to obſerve. In the firſt Figure I ſuppoſed divers Rays to be carried ſucceſſively along the ſame right Line to a refracting Surface, and there ſome to be gradually a little more refracted than others. Which if it were conceived to be done, it would abundantly follow, that the Rays ſo refracted, if they were afterwards intercepted by any opake Body, as Paper, they would there paint a ſmall bright Line. Now although the Rays coming from any Star do not all proceed in the ſame right Line, however what is equivalent, they may be looked upon as parallel, and by Reaſon they are by the convex Lens made to converge before they arrive at

21. And it is applied to the Deſcription of Refraction delivered at the 1ſt Figure.

the Prifm, this does not deftroy the Analogy, but greatly confirms it. Becaufe for every one proceeding in the fame right Line, you ought only to conceive fo many Pencils of Rays, which all have the fame Axis, and the fame Point of Concourfe; and thus of thofe Pencils, fome are more refracted by the Prifm than others, fo that their Points of Concourfe, or Foci, which before coincided, now every one fall feparate making a right Line. And therefore that the Axes of the Pencils, which coincided fo long, fuppofe with the fucceffive Rays, until they arrived at the Prifm, and there by the various Refraction they are made to diverge, that they proceed to the Foci of the Pencils lying in a ftraight Line.

22. A varied Circumftance again applied to the fame Defcription.

IF you place the Prifm nearer to the Planet *Venus* than the Lens, that the Rays may be firft tranfmitted thro' it, and then made converging by the Lens, you will fee as before the fame fmall Line, although lefs confpicuous, and more difficult to be found. Now

in this Specimen, ſince all the Rays come parallel, if they were equally refracted, as they thus paſſed through the Priſm, they would remain parallel afterwards, until they fell upon the Lens, and in that they would therefore be ſo refracted, as all would thence forward proceed to the ſame Point, and ſo a lucid Point would be ſeen. Wherefore ſince inſtead of ſuch a Point a Line appears, it muſt be concluded, that all the Rays are not equally refracted.

IF any one ſhould now object, that in Refractions there is indeed an Inequality, but that it is contingent and not ariſing from a previous Diſpoſition of the Rays, or by any certain Laws, I anſwer, that the above mentioned Image of the Sun, if it became oblong by Rays refracted according to no Law, it could not be terminated in its Length by ſtraight Lines, as has been ſhewn at the fifth Figure. Beſides it ought not at all to be oblong, but to be formed in its middle Part in the Shape of an Orb more ſplendid, and by a ſenſible Termination to be diſtinguiſhed

23. That in the produced Experiments the Refractions are not made unequal by chance, or by any other Cauſe than an unequal Refrangibility.

ed from the weaker, erratic Light dispersed every where about. Just as the Sun appears, when he is almost obscured by the Clouds, or as his Image is seen, when transmitted through a Glass Plate terminated by parallel Planes, and lightly o'erspread with one's Breath or Smoke, that the Light in being refracted may be a little disturbed. Add

Fig. 10. to this, if (*Fig.* 10.) two similar Prisms A B C and *a b c* be placed by one another parallel in respect to their Length, with their plane Sides A C, and *a c*, as also B C and *b c* parallel, and if the Sun shines through both of them into the Place Z, where an opake Body is directly opposed to the Light; his Rays however being first transmitted through the circular Hole F. The Light incident on the said Z will appear distinctly circular, not otherwise than if it directly tended from F, the Prism being not at all interposed. It must therefore be acknowledged, that the Refractions of both Prisms conjointly are regular, and therefore also the Refractions of either of them. *Viz.* those Rays alike inci-

incident are not all refracted equally in the first Prism A B C, nor in the second *a b c*, yet since the Inequality of the Refraction is not contingent, but arises from a previous Disposition of the Rays, therefore although various Rays are variously refracted, yet of the same Ray there will be the same Quantity of Refraction in both Prisms, and as much as it is incurvated by the first A B C, so much will it be incurvated by the second *a b c*, whence any Ray, however refrangible it is, after it has emerged out of both Prisms, will become parallel to it self, before it had been incident on them. And therefore since all tend towards the same Parts, to which they would freely have tended, if they had not been intercepted by the Prisms, it necessarily follows, that they exhibit the same circular Image at Z, which they would exhibit, had they freely tended thither. But if the oblong Image made (as has been said) by the Refraction of one Prism only, did acquire its Figure by Rays divaricating by no certain Law, but refracted scateringly by chance here and

there, when the Refractions were doubled by two Prisms, the Errors also of the Rays would become twice as many and also twice as great, and from thence the Image at Z would become much more oblong, which yet by Experience is found to be contracted into an Orb.

SOME may perhaps suspect, that the Termination of the Light, or the confine of the quiescent medium, may cause the Diversity of Refraction; but to this Suspicion a Remedy is at hand; *viz.* by making (as in *Fig.* 3.) the Light to be terminated by the hindermost Part of the Prism only, that it may not become bounded by the Shadow before it shall be refracted. And therefore that no Suspicion be raised about the various Thickness of the Glass, its Refraction may be tryed at different Thicknesses, by moving the Prism transversly with a parallel Motion near the Ingress of the Light, so that the Light at first may be transmitted at its Edge, then at thicker Parts of it; and in every Case the Appearance of the

Fig. 3.

the Colours will be alike. Nor does it much import, whether the Hole, through which the Light enters, is broader or narrower; for from thence nothing elſe follows, beſides the Augmentation or Diminution of the Light exhibiting the Colours, and ſo great a Dilatation or Contraction of the Image, as there is of the Hole.

FROM the Experiment of the two parallel Priſms already deſcribed, it is alſo manifeſt, that this dilating of the Image in Length does not ariſe from the ſpreading or ſplitting of any the ſame Ray into diverſe diverging Rays, for they by another ſpreading or ſplitting, in their Paſſage through the ſecond Priſm, ought then to be reſolved into a far greater Number and more diverging Rays. Moreover to all theſe Objections is oppoſed the Experiment, where the latter Priſm is not placed parallel to the former, but perpendicularly tranſverſe. For in that Caſe, if the former Priſm dilated the Image in Length from any other Cauſe than the different Refrangibility of different

Rays,

Rays, then the latter Priſm by a tranſverſe Refraction ought to dilate that oblong Image in Breadth, and ſo would form a quadrilateral one. But upon trying the Experiment, the Thing came out otherwiſe, *viz.* the Image not being dilated in Breadth, but only rendered oblique by the leſs Refraction of the red Extremity than of the violet, as is to be ſeen at *Fig.* 11. where the Image P T by the Refraction of the ſecond Priſm is transferred to *p t*. From what has been ſaid, I believe, it more than ſufficiently appears what I propoſed at firſt to demonſtrate: But becauſe the Agreement of many Things affords Delight to the Underſtanding, and often begets a firmer Aſſent, than the Teſtimony of a ſingle though the moſt ſcientific Argument, it will not be foreign to our Purpoſe if we briefly introduce another Sort of Experiments a-kin to the preceding ones.

Fig. 11.

24. Other Experiments a-kin to the preceding are touched upon. *Fig.* 12.

IN *Fig.* 12. let F be a very ſmall Hole, through which the Light of the Sun may be tranſmitted; and at a Diſtance taken at Pleaſure, let be placed the Priſm

Priſm A B C, through which the Rays may paſs refracted, as I have explained in the former Experiments; then with your Eye placed behind, you will ſee an oblong Image T P of the Hole F; whoſe Length compared to its Breadth, will be ſo much the greater, as the Hole F ſhall be made narrower, and thence it appears, that of the Rays ſome tending to the Eye by H, as if they had flowed from P, are more refracted, than others tending by I, as if they had come from T, and the Rays not otherwiſe entering the Eye, than if they had flowed from the oblong Space P T, it neceſſarily follows, that that long Space muſt appear illuminated.

BUT Care muſt be had, that the Aperture of the Hole F is not ſo large, as to hurt the Eye by the admiſſion of too much Light; nay it ought not to be greater, than that you may ſee through that Hole with your naked Eye a Particle of the Sun like a lucid Point diſtinct, and without any Circumradiation. But if the Light of the Sun be thought

thought too much for this Experiment, the Light tranſmitted from the Clouds may be ſufficient, provided the Diſpoſition of your Eye be ſuch, as you may diſcern the Hole diſtinct without any incompaſſing Rays, before you interpoſe the Priſm, otherwiſe you will not ſee its Image diſtinct, nor deduced into a due Length. Beſides you may alſo obſerve, if you look at a Thread through a Priſm, for the Thread will appear much broader, when it is placed in a Situation parallel to the Length of the Priſm than when in a tranſverſe one. But that I may comprehend all in one, if you behold a Star of the firſt Magnitude through the Priſm, its Image will alſo be ſeen long. But ſince the Rays of the Stars are eſteemed as parallel, if they all were refracted equally, they would alſo remain parallel after they iſſued out of the Priſm, and ſo entering the Eye would make an Image altogether like the Star, or a lucid Point, not at all long; juſt as it happens, when the Star ſends parallel Rays directly into the Eye. You therefore ſee, that parallel

Rays

Rays refracted by plane Surfaces become inclined, whence it necessarily follows, that they suffer an unequal Refraction. But it may be remarked by the by, that a Telescope, if you think fit, being first applied, both that a sufficient Quantity of Light may be transmitted to the Eye, and that the Scintillation, wherewith the fixt Stars are wont as it were to be crowned, may be lessened, and a Prism then being interposed, you will see the whitish Line more distinct than before, with a Breadth scarcely discernible. These few things being declared concerning the different Refrangibility of Rays, the meaning whereof will appear more fully in what follows, when we come to treat of [a] Colours: It remains, that the Quantities and Measures of Refractions be now determined.

[a] *Viz.* in the Philosophical Transactions No. 80, *&c.* and more fully in his Treatise of Opticks.

SECTION

SECTION II.

Of the MEASURE *of* REFRACTIONS.

25. Of the measure of the Refraction of Rays of a given kind from any given Incidence. *Fig.* 13.

THE Ancients determined Refractions by the Means of the Angles, which the incident and refracted Rays made with the Perpendicular of the refracting Plane, as if those Angles had a given Ratio. For Example, in *Fig.* 13. let I H be a refracting Plane, to which the Line D C E perpendicularly insists at one of its Points C, and in this Point C let any Ray A C be incident, and let it be refracted to R: the refracted Ray C R being supposed to lie in the Plane A C I, that is perpendicular to the refracting Plane; the Antients supposed, that the Angle of Incidence A C D, the Angle of Refraction R C E, and the refracted Angle R C F are always in a certain given Ratio; or they rather believed it was a sufficiently accurate Hypothesis, when the Rays did not much divaricate from the

the Perpendicular. So in Glaſs they made the Angle of Refraction to be about tripple of the refracted Angle. But this eſtimating of the Refractions was found not to be ſufficiently accurate, to be made a Fundamental of Dioptricks. And [a] *Cartes* was the firſt, that thought of another Rule, whereby it might be more exactly determined, by making the Sines of the ſaid Angles to be in a giving Ratio. In *Fig.* 13. if with the Center C, and any Diſtance A C a Circle be deſcribed cutting the aforeſaid Rays in A and R, and from thoſe Points to the Plane's perpendicular D C E, the nomals A D and R E be drawn, the Proportion of A D and R E will be perpetually the ſame. The Truth whereof the Author had demonſtrated not inelegantly, provided he had left no room to doubt of the phyſical Cauſes, which he aſſumed. And as alſo ſome have examined this with Inſtruments accurately made for that Purpoſe, and have found it (as to Senſe) exactly agreeing with the Truth; we do *Fig.* 13.

[a] Of this ſuppoſed Diſcovery of *Cartes*, ſee the Note at Page 13.

not

not ſcruple to receive it as a Fundamental, but with this Caution only, that ſince he affirmed it indifferently of any Sort of Rays, we only affirm it of each of their Kind conſidered apart, by laying down, that the Sine of Refraction of equally refrangible Rays are as the Sines of Incidence. Let us conceive ſome Kinds of Rays to flow along the Line A C in *Fig.* 14. to the Point C, and there to be refracted by the Surface I H, ſuppoſe the mean refrangible Rays into C R, the leaſt refrangible into C T, and the moſt refrangible into C P, and innumerable others of intermediate Degrees more or leſs refrangible to be diffuſed thro' the whole Space T C P. Now if D C G be drawn perpendicular to the refracting Plane I H, and with the Center C, and any Diſtance A C, let a Circle be deſcribed as before, cutting the ſaid Rays in A, P, R, T, and from theſe Points let be fallen the perpendiculars A D, P G, R E, T F, for the Sines of the Angles A C D, P C G, R C E, T C F, I ſuppoſe that however the Rays fall, yet it will be always

Fig. 14.

A D

A D to P G in the ſame Ratio, which being once known, you have a Rule for meaſuring the Refraction of the moſt refrangible Rays, falling upon the ſame Surface at any Angle; and ſo it will always be A D to T F, in the ſame Ratio, which being known, you have a Rule, whereby you may determine in any Incidence the Refraction of the leaſt refrangible Rays: And the ſame Thing may be conceived of the Ratio of A D to R E, and to the Sine of any intermediate Sort.

BUT moreover, ſince the Sines P G, R E, T F, and the reſt have a given Ratio to the Sine A D; they will have alſo a given Ratio to one another, and therefore if from one Obſervation you know the Proportion of the Sines P G, R E, T F, and the reſt belonging to the Rays refracted from the ſame Incidence, you will thence have a Rule, whereby having given the Sine of Refraction of any Sort of Rays, and thoſe falling any how on that Surface, you may diſcover the Sines of all the reſt flowing from the ſame Incidence, although what

26. Of comparing the Refractions of Rays of different Sorts.

what their Incidence is, be not known. Therefore that the Ratios of thoſe Sines may be inveſtigated, it is proper firſt to find, in ſome Sort of Rays, the Proportion of the Sine of Incidence, to the Sine of Refraction, then that there be determin'd the Proportions of the Sines of Refraction, for Rays of different Sorts, incident with the ſame Angle.

27. To compare the Sines of incidence and Refraction, there ſhould be uſed the mean Sort of Rays.

To compare the Sines of Incidence with the Sines of Refraction, it will be proper to uſe the mean Sort, *viz.* that Sort of Rays, which exhibit a green Colour, or rather a Colour between Green and Blue: For I believe thoſe, who hitherto have meaſur'd Refractions, (whether it was done in order to confirm the Hypotheſis of *Cartes*, or for other Reaſons) I believe, I ſay, they accommodated their Meaſure to the Middle of the refracted Light; that is, if we regard the Space occupied by the Colours, to the Confine of Green and Blue. Or if we regard the Quantity, to the Middle of the Green: And beſides, that Point ſeems to have been looked upon as the principal Focus of

of convex Glaſſes, to which the intermediate Sort of Rays converged. And alſo, if at any Time we ſpeak indiſtinctly of Rays, as has been hitherto the Cuſtom with the Writers in Opticks, the middle Sort may be more commodiouſly uſed for all than any of the extreme Sorts.

MOREOVER, ſince perhaps there may be required a more accurate Examination of the ſaid Rule of *Cartes*, than has hitherto been made, whilſt the various Refrangibility of Rays was unknown, I ſhall firſt declare after what Manner, that may not be inconveniently done. Becauſe the refracting Surfaces of a tranſparent Fluid may be eaſily inclined to any given Angle, which is not permitted to a Solid, Fluids have been made Uſe of to this Purpoſe, but by an Inſtrument more operoſe than was neceſſary, and poſſibly more obnoxious to Errors, than if it wanted all that Apparatus, except a Beam to which a Veſſel full of Water is faſtened; let therefore (in *Fig.* 15.) H K be a Beam of Wood Two or Three

28. The Manner of exploring the Proportions of thoſe Sines.

Fig. 15.

Yards, or more long, ſufficiently thick, that it may not in the leaſt bend by its Length or Weight, quadrilateral, rect-angular, and ſtraight, with oppoſite Sides exactly parallel; then let two ſmall Pieces, H I and K L be erected at right Angles, upon one of its Sides; K L near to one of its Ends, and H I about Four Inches diſtant from the other, the Length of which Two Pieces, let be Three or Four Inches, but their Breadth Two or Three: Then let be taken a cylindrical or priſmatic Veſſel C F, Two or Three Inches broad, but Four or Five long; let its Baſe be faſtened upon the Piece H I, by ſome hard and tenacious Cement, and let it be fixed in that Situation by the Means of the Beam H K, produced beyond the ſaid Piece H I; then let its Bottom, as alſo the Piece be bored in the Middle with the ſmall Hole F, ſuppoſe the tenth Part of an Inch broad; and right againſt this Hole, let be marked in the other Piece the Point R, that may be at the ſame Diſtance from the Beam, with the Center of the ſaid Hole, in order that the Line F R, drawn

drawn through the Center of the Hole to R may be parallel to the Length of the Beam. *Laſtly*, Let be taken a Glaſs Plate, plain, ſmooth, and uniformly thick, and let it be applied to the Side of the Piece H I, which is towards the Veſſel C F, over the Hole F; and let it be ſo faſtened with Cement, that the ſaid Veſſel may hold the Water it is filled with; and let Trial be made with a Square, whether the Glaſs Plate is perpendicular to the Beam; which if it happens not to be, let its Situation be corrected, till it is exactly perpendicular; for which Purpoſe it is fit, that the ſaid Glaſs Plate be Three or Four Inches broad and long, whereby a better Judgment may be made of its Situation. This Inſtrument thus framed, and the Veſſel CF above half filled with Water, let it be ſo placed in the Sun's Rays, that they being refracted on the upper Surface of the Water, may emerge perpendicularly at the Hole F, and may proceed ſtraight forwards, towards the Piece KL, the red Rays falling at T, the Purple at P, and the Green, or the Confine of Blue

and Green at R. It is convenient, that the ſaid Piece K L be painted white, or covered with a white Paper, whereby you may judge the better of the Colours. But in the mean while, with ſome large and exactly made Quadrant *e k r*, let be ſought the Inclination of the Beam H K to the Horizon, and you will have the Angle of Refraction *e k r*, and its Sine *e r*; then let the Height of the Sun be immediately taken, and its Complement to 90 Degrees A *k* D will be the Angle of Incidence, and A D its Sine: Which Sines being compared together, and the Experiment being repeated at different Heights of the Sun, it will appear, whether the Proportion of the Sines is always the ſame; but if you would, that divers Experiments be made at the ſame Time, or at a leſſer Incidence than is the Compliment of the greateſt Height of the Sun, inſtead of the Rays flowing directly from the Sun, you may uſe reflected ones.

29. The Manner of exploring the refracting Force of any Solid encompaſſed with Air.

SINCE it has been ſufficiently diſcovered, that the ſame Proportion of

the

Plate 3^d.

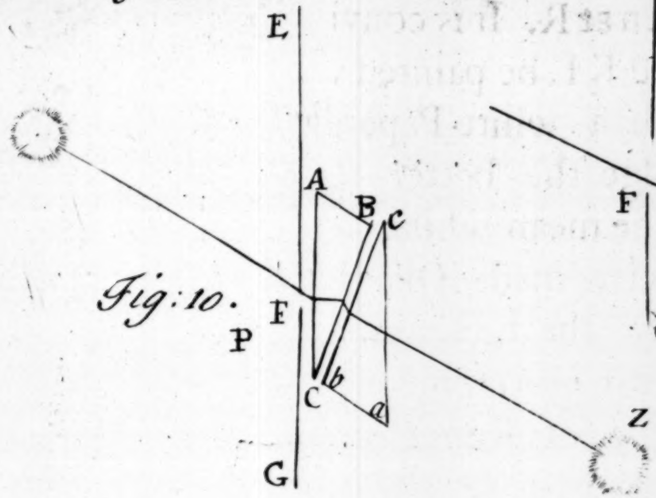


Fig: 10.

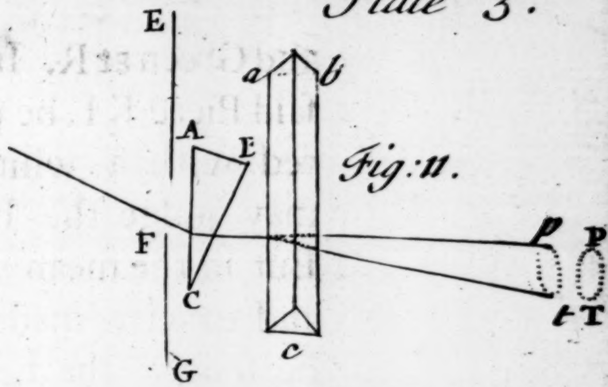


Fig: 11.

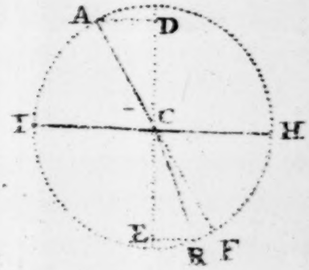


Fig 13.

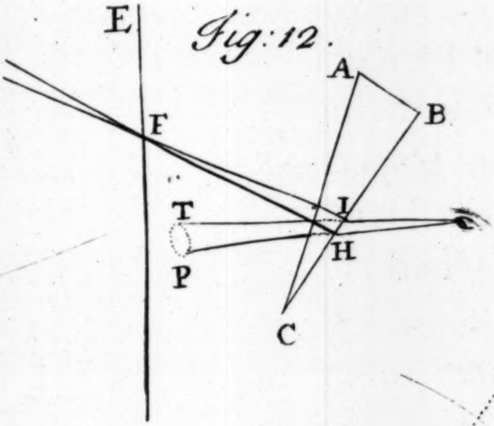


Fig: 12.

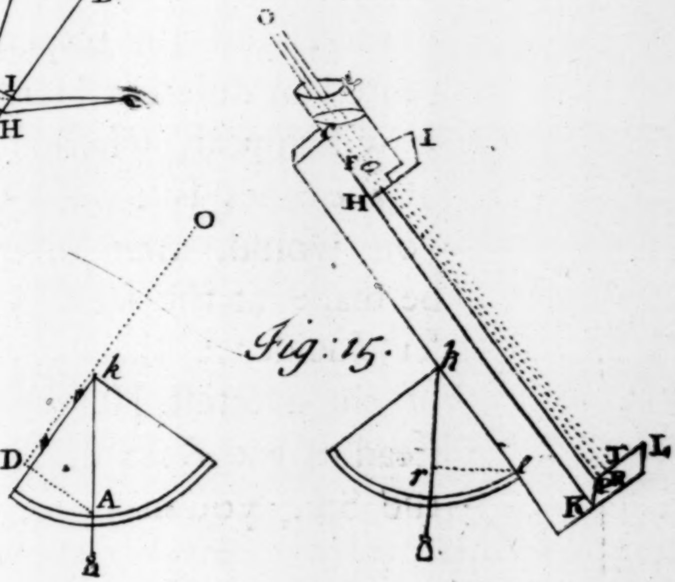


Fig: 15.

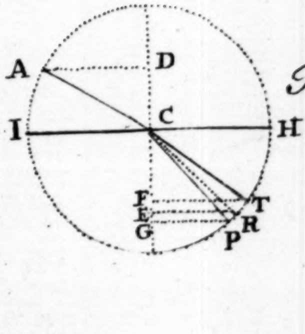


Fig: 14.

the Sines of Incidence and Refraction does perpetually belong to any one Sort of Rays, however incident on any the ſame Surface: let it be propoſed to find that Proportion at a Surface ſeparating any given Mediums, and that by one Experiment. If the Air be one of the given Mediums, and any Liquor the other, the Inſtrument juſt now deſcribed may not incommodiouſly be uſed. But if the other Medium be a Solid, the Thing is expeditiouſly done at the 16th *Diagram*. For the explaining of which, we ſhall premiſe the two following *Lemmas*. *Fig.* 16.

LEMMA I.

IN *Fig.* 16. *let* A B C *be a Priſm made of any tranſparent Subſtance, whoſe Axis let be parallel to the Horizon and perpendicular to the Sun's Rays, and beſides let its Poſition be ſuch, as it may equally refract the ſaid Rays* O X *entering at* X *and going out at* Y. But how that may be done, was taught at N°. 10. *Now I ſay, that the Angle of Refraction, made at either of the refracting Surfaces as*

A C, *is equal to half the vertical Angle of the Prism* A C B.

For at the Point of Incidence X, let be erected the perpendicular H X, then H X Y will be the Angle of Refraction at the Surface A C: Moreover let C I be set full perpendicular on the Ray X Y, and it will bisect the Angle Y C X, because the Triangle Y C X (on account of the Equality of Refraction in X and Y) is Isosceles; I therefore say, that the Angles H X Y and I C X are equal. For the Angles A X Y = ang. X I C + I C X (by 32. 1. *Elem.*) but the Angles A X H and X I C are right; therefore the Remainders H X Y and I C X are equal, Q. E. D.

LEMMA II.

Again if the incident Ray O X *and emerging one* Y N *be indefinitely produced, meeting in* G, *and besides, if any right Line* K L *parallel to the Horizon be crossed by these Rays, making the Triangle* G K L, *and when the refracted Ray* Y N *tends upwards, if the Sum of*

of the Angles L K X *and* K L Y *be taken, or their Difference, when that Ray* Y N *tends downwards: I say, that the half of that Sum or Difference together with the Angle of Refraction* H X Y *will be equal to the Angle of Incidence* H X G.

FOR the said Sum or Difference is equal to the Angle N G K (by 32. 1. *Elem.*) that is, to the Angles G X Y + G Y X, and since the Triangle G Y X is Isosceles, the half of the said Sum or Difference will be equal to the refracted Angle G X Y, which makes with the Angle of Refraction Y X H the Angle of Incidence. Q. E. D.

THESE Things being premised, the *Problem* is thus done; in the first Place let be measured the vertical Angle of the Prism A C B, and its half will be the Angle of Refraction. Then the Prism being disposed in the aforesaid Position, through which the Rays entering the Hole F may be transmitted, let by the means of a large and accurate Quadrant M N P Q (the Distance of whose

whose Sights M and N being one Foot at least) be found the Angle Y L K or P *k* Q, which the refracted Rays Y M N make with the Horizon, by causing the mean refrangible Rays to pass through the Sights M and N at the Distance of ten or twelve Feet from the Prism, and at the same time let be observed the Heigth of the Sun X K L. Which two Angles let be added, if the refracted Rays Y M N tend upwards, as is described in the *Scheme*, otherwise let the less be taken from the greater; and half the Sum or Difference together with the Angle of Refraction found before will be the Angle of Incidence, as is manifest by the second *Lemma*. Lastly from the Angles of Incidence and Refraction being thus given, their Sines are given. Q. E. F.

30. An Example in the Refraction of a certain kind of Glass.

So in a certain Glass Prism I measured its greatest Angle A C B, and found it to be 63 deg. 12 min. whose half H X Y is 31 deg. 36 min. and its Sine 5240, the Sine of 90 deg. being made 10000. Then since the Sun's Height O K L was observed to be 14 deg.

deg. 4 min. the other Angle M L K made by the Ray Y N tending to the middle of the green was 30 deg. 52 min. whose Sum is 44 deg. 56 min. and its half Y X K 22 deg. 28 min. which together with the Angle of Refraction H X Y makes 54 deg. 4 min. the Angle of Incidence, whose Sine is 8097. Lastly by comparing the Sines now discovered, that their Proportion may be had in the least Terms, I found it to be as 11 to 17 nearly. Wherefore it may be laid down as a general Rule, that of Rays exhibiting a green Colour, the Sine of Incidence, out of Air into any Glass of equal Refraction with this Prism, is to the Sine of Refraction, as seventeen to eleven.

BY measuring in like manner the Refraction of Rays exhibiting a Colour between green and blue, 45 deg. 8 min. was found for the double of the refracted Angle, whose half 22 deg. 34 min. together with the Angle of Refraction 31 deg. 36 min. gives the Angle of Incidence 54 deg. 10 min. And its Sine 8107 is to the Sine of Refraction

31. The Conveniency of the aforesaid Method. tion 5240, as 82 to 53 nearly. But the Conveniency of this Method in measuring Refractions may be guessed at from this, that there is here no Occasion for any Instrument, besides a Quadrant and the Prism, whose Refraction is desired; that the Refraction, whilst it is rendered double at X and Y, may be thence more certainly measured; and that it is very easy to dispose the Prism in the desired Situation, as has

No 10. been shewn above at No 10. And farther that a small Error in the desired Situation is scarcely of any Account, whilst as to Sense the refracted Angle M G K will not thence be changed; as will appear upon Tryal. For that Angle is here * the least of all, and of Quantities generated by Motion, when they become the greatest or the least, that is in the Moment of their Regress, the Motions are for the most part infinitely small. So for Example,

Fig. 17. in *Fig.* 17. If with the Center C be described a Circle *l* L *l*, and without

* This will appear from what our Author demonstrates hereafter at Sect. III. Prop. xxv.

it

it be taken any Point G, and there be drawn G I C, and the perpendicular G K erected. Then if it be conceived, that the Point *l* be moved uniformly in the Circumference of that Circle, thro' which Point a certain ſtraight Line G *l* turning on the Center G, may continually paſs, it is manifeſt, that the greater the Angle C G *l* is, or the leſs the Angle K G *l*, the leſs will be the angular Motion of the Line G *l*, and when the Angle C G *l* becomes the greateſt, or the Angle K G *l* the leaſt, that is, in the Moment of Regreſs (the right Line G *l* then touching this Circle in L) its Motion will be infinitely ſmall, and as to Senſe none, and a ſmall Error from the Point of Contact L will produce no ſenſible Variation in thoſe Angles K G L and C G L. And nearly after the ſame Manner a ſmall Convolution of the Priſm will not at all alter the Angle M G K (*Fig.* 16) when it its the leaſt, or its Complement the greateſt. But if the Priſm be diſpoſed in any other Situation than is here deſcribed (as when the Rays entering perpendicularly are only refracted *Fig.* 16.

ted

ted at their going out) a very ſmall Error from that deſired Situation will much alter the refracted Angle, and ſo the Experiment would be much more liable to Uncertainty and Errors.

32. A *Rule* to find the Refraction of Mediums contiguous to one another, whoſe Refractions when contiguous to the Air are known.

FOR the greater Variety in this matter, becauſe there are given ſome Caſes, where the Refractions cannot be meaſured by the Ways hitherto deſcribed (as when the Refraction is made out of Glaſs into Cryſtal, out of Water into Glaſs, or out of one Liquor into another) and that there may be no refracting Surface, whoſe Refraction cannot be inveſtigated, it is thought proper to propoſe the following Problem.

The Refractions being given, which are made by two Mediums, contiguous to a third, to find their Refraction, when contiguous to one another.

Fig. 18.

2 *Fig.* 18. LET the two Mediums propoſed be A and B, the Refraction of whoſe terminating Surface is ſought, and let C be a third Medium, the Re-

fractions of whose Surface contiguous to A and B are given. And let the Sine of Incidence to the Sine of Refraction out of the Medium C into the Medium A be, as I to R, and the Sine of Incidence to the Sine of Refraction, out of the same Medium C into the other Medium B, as *j* to *r*; I say, that it is $I \times r . R \times j$: : Sine of Incidence to the Sine of Refraction out of the Medium B into the Medium A.

FOR Example, let be proposed the Investigation of the Refraction out of Water into Glass, the Refraction out of Air into both being given; and let the Sine of Incidence out of Air into Glass be to the Sine of Refraction, as 17 to 11, and the Sine of Incidence out of Air into Water to the Sine of Refraction, as 4 to 3. Wherefore by multiplying reciprocally these Sines, it will be as 17×3 to 11×4, or as 51 to 44, so the Sine of Incidence out of Water into Glass to the Sine of Refraction. And after the same manner the Refraction out of Air into any other proposed Mediums being known, you

you may find their Refractions amongst themselves, and on the contrary.

33. The Demonstration of the *Rule*.

Fig. 18.

BUT the Demonstration of this must not be omitted, for which is premised the following *Lemma*. If the two proposed Mediums A and B in *Fig.* 18. be conceived to be terminated by parallel Planes, contiguous, and encompassed by the said third Medium, as suppose Air, and any Ray O N falling obliquely at N be refracted first to M, and next to L, and it emerging proceeds to K, I say the incident Ray O N is parallel to itself, as emerging L K. The Truth whereof appears indeed by Experience: For let the Medium A be supposed Glass, and the Medium B to be Water, and the third encompassing Medium to be Air; and let the Surface M R of the Glass Plate A be lightly smear'd with the Water B, and let it be placed parallel to the Horizon, that the Water may be of an equal Thickness; which being done, you will see, that the Rays transmitted through both the Mediums A and B, will tend to the same

ſame Parts, towards which they would tend directly from the Sun.

THIS being premiſed, let be erected jNr, HMG and RLI perpendiculars at the refracting Points N, M and L; it is therefore j to r, as the Sine of the Angle ONj, to the Sine of the Angle MNr, or NMH; and by multiplying the antecedent Ratio by I, it will become $I \times j$ to $I \times r$, as the Sine of ONj to the Sine of NMH. Moreover, it is I to R, that is, $I \times j$ is to $R \times j$, as the Sine of the Angle KLI to the Sine of the Angle MLR, that is, as the Sine of the Angle ONj to the Sine of LMG. Now, by permuting the Terms of both the Proportions, it will become $I \times j$. Sine ONj :: $I \times r$. Sine NMH, and $I \times j$. Sine ONj :: $R \times j$. Sine LMG. Wherefore, by Equality of Ratio it is $I \times r$. $R \times j$ :: Sine NMH. Sine LMG. Q. E D.

FROM theſe thus ſhewn, a no unuſeful *Problem* ariſes, whereby the Refractions of Fluids may be meaſured by the ſame Means, as is ſhewn of Solids at

34. The Manner of meaſuring the Refractions of Solids accommodated to Fluids.

at *Fig.* 16, without uſing the Inſtrument HLK, that is deſcribed in *Fig.* 15. For Example, let a Priſmatic Veſſel be made of Glaſs Plates, connected in the Shape of a Wedge, whoſe Edge or vertical Angle let be about 86 or 90 Degrees. But you muſt have known the Quantity of that Angle by a very exact Meaſure, and always put the Sine of its half for the Sine of Refraction. Which being done, when the refractive Power of any Liquor is deſired, let the Veſſel be filled with that Liquor, and placed in ſuch a Situation, that the Edge made by the Concourſe of the refracting Planes may be parallel to the Horizon, and perpendicular to the ſolar Rays, and that thoſe Rays tranſmitted through the ſaid refracting Planes may ſuffer equal Refractions at their Ingreſs and Egreſs. And by the Help of a Quadrant, as was ſhewn at *Fig.* 16, let be found the Angle of Incidence, whoſe Sine to the aforeſaid Sine of Refraction will be, as the Sine of Incidence to the Sine of Refraction out of Air into the propoſed Liquor.

FOR Inſtance, that I might know the Refraction of Water, I procured a wooden Priſm to be made, ſuch as is A B *c* in *Fig.* 19, whoſe Angle A C B, which I deſigned for the vertical one, was right, and the other two half a right one, and I cauſed the refracting Planes A *c* and B *c* to be bored through the middle by the Hole F parallel to the Baſe A *b*, by which Hole the Light is to paſs, and the third Plane to be bored in G, till a Way is made tranſverſly to the Hole F. Then taking two Glaſs Plates, which a broken Looking-Glaſs furniſhed me with, I fixed one over the very middle of the Plane B *c* by Cement, and the other over the middle of the other Plane A *c*, that the Paſſage F might be cloſed on both Sides. Then I poured Rain Water thro' the Orifice G into the hollow Space, and ſhut it with a Stopple made of Cork. And ſo the Water, included between two Glaſs Plates, which were inclined at right Angles, ſerved inſtead of a Priſm of Water having a right Angle. But that theſe Plates contained

35. The Refraction of Water, as I myſelf have meaſured it, brought as an Example of this thing. *Fig.* 19.

exactly a right Angle, I knew by the means of a Square; whose half 45 Degrees therefore is to be reckoned the
Lemma 1. N° 29. Angle of Refraction. This Prism I then so placed to the Ingress of the Light into a dark Room, that the Quantity of Refraction on both Sides might be the same, and from the Sun's Height, and the Inclination to the Horizon of the refracted Rays exhibiting a green Colour, I found the refracted Angle to be 51 deg. 16 min. whose half 25 deg. 38 min. together with the Angle of Refraction 45 deg. will give the Angle of Incidence 70 deg. 38 min. But the Sines of these Angles 70 deg. 38 min. and 45 deg. are 9434, and 7071 in respect to the Sine of 90 deg. 10000. The Ratio of which Numbers is indeed a little less than *Cartes*'s of 250 to 187, and a little greater than 4 to 3, *viz.* 4,002 to 3, which however differs so little from the Ratio $\frac{4}{3}$, that the Error would have been insensible, if I had put it as 4 to 3, and that chiefly since the Refraction of Water remains not always the same, but suffers something from the Change of Colour, and

and assumes various Degrees of Density. Which same Thing happens to the ambient Air, which also is not only variously incrassated by Vapours, but likewise more closely (the Weight of the Atmosphere being increased) or more laxly compressed. Add that there are different Densities of Waters springing in different Regions of the Earth, or by the Force of the Sun converted into Vapours, and thence into Rain; and their internal Dispositions to refract are different, arising from various mineral Tinctures, which they extract from subterraneous Places, and from Exhalations variously dense and copious, which are raised aloft together with the Vapours.

THE Truth of this *Problem* of the Measure of the Refraction of Fluids thus solved, will appear by shewing, that the Quantity of the Refraction in this Prism compounded of Water and Glasses is the same, as it would be, if the Glass was taken away, and the Water alone remained incompassed with the Air. Let therefore in *Fig.* 20, A B C be

36. The Demonstration of what has been said.

Fig. 20.

be a Priſm made of Glaſs Plates A C *f d* and B C *f e* (as has been ſaid) and filled with Water *d f e;* and let it be conceived, that D E F is a Priſm of Water immediately incompaſſed by the Air, and altogether like to the aqueous one *d e f* incloſed in Glaſs, and ſimilarly poſited ; and let parallel Rays O N, O X fall upon both, whereof one O N refracted in N, M, L and K tends to H, but the other O X refracted in X and Y tends to Z. I now ſay, that the emerging Rays K H and Y Z will be parallel, and therefore that in both Priſms the whole Quantity of Refractions will be the ſame. For in *Fig.*
Fig. 18. 18, if the Ray *o m* parallel to O N fall on the Glaſs Plate A, and emerges in *l k*, it is known, that the Ray *l k* will be parallel to *o m*, that is to O N and L K, and ſince *l k* and L K are parallel, *m l* and M L will be alſo parallel; whence appears the *Propoſition*, that the Quantity of Refraction out of Air into any propoſed Medium is the ſame, whether the Rays enter immediately that Medium out of Air (as is done at *o m l*) or they firſt pervade ano-

ther

ther Medium interpoſed, and terminated by parallel Planes, (as is done at O N M L) and on the contrary. And the ſame thing is to be underſtood, when inſtead of Air any other Medium is uſed. Wherefore in *Fig.* 20. ſince the parallel Rays O X and O N fall upon the Priſms D F E and A C B ſimilar and ſimilarly poſited, the Quantity of the Refraction out of Air into Water will be the ſame, whether the Rays immediately enter, as is to be ſeen at D E F, or firſt pervade the Glaſs Plate A *d f* C, that is, the Ray X Y once refracted will be parallel to M L twice refracted; and for the ſame Reaſon, ſince X Y and M L are parallel, the emerging Rays Y Z and K H will be alſo parallel. Wherefore ſince the incident and emerging Rays are parallel, the whole Refraction of both Priſms will be the ſame. And therefore ſeeing an aqueous Priſm contiguous to the Air cannot be made by Reaſon of the Fluidity of the Water, in its Stead may be uſed a Glaſs Priſm filled with Water. Q. E. D. *Fig.* 20.

 AND

AND thus is ſhewn a general Method, whereby the Refractions, out of Air into any propoſed Medium, may be determined, which is very eaſy, and little liable to Errors, eſpecially if the Angle of the Priſm is large, and exactly known, the Quadrant large and accurate, and the Obſervation made far behind the Priſm, where the Colours being much dilated, are the more eaſily to be diſtinguiſhed. And beſides, ſince the Refraction between Air and the propoſed Mediums, are thus determined by Experiments: A Rule is laid

N°. 32. down, N°. 32, whereby the Refractions of the ſame Mediums, contiguous to one another, may be diſcovered. Which is ſufficient to have ſhewn in illuſtrating the firſt Caſe, of meaſuring Refractions, when the Proportion of the Sines of Incidence and Refraction is ſought in the ſame Sort of Rays.

37. The Refractions of different Sorts of Rays are compared, and the Difference of the greateſt Refrangibilities is inveſtigated.

THE other Caſe is now to be proſecuted, where the Refraction of Heterogeneal Rays are to be compared. But that

Fig: 16.

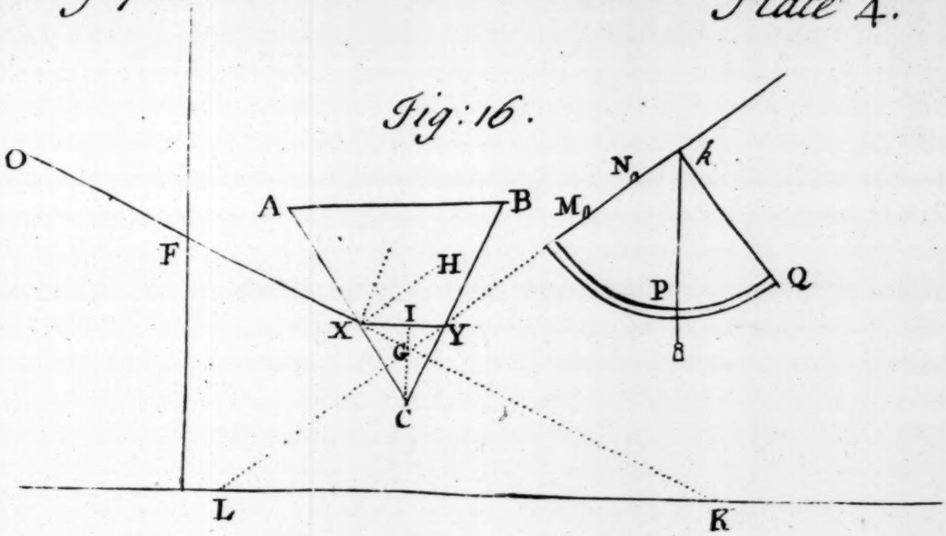


Fig: 17.

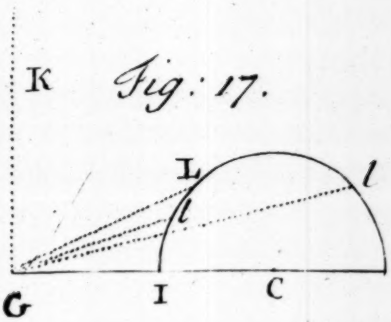


Fig: 19.

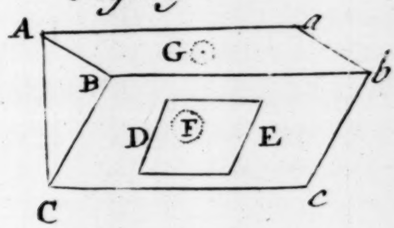
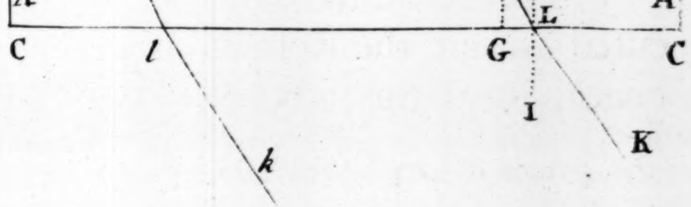


Fig: 18.



that the Sine of Refraction of any Sort of Rays is to the Sine of Incidence, in a certain given Ratio, you may try, by measuring the Refractions of every remarkable Sort Incident separately, according to various Obliquities upon some refracting Medium, as upon Water (at *Fig.* 15.) stagnating in a Vessel, or upon Glass Prisms, whose vertical Angles are of different Magnitudes. For by one Prism you may find the Proportions of the Sines to every Sort of Rays, as was shewn at *Fig.* 16, then by other Prisms (or by the other Angles, whether greater or less of the same Prism) you may discover whether the same Proportions are observed in other Obliquities. And so (Observations being most accurately made) it will at the same Time appear, that the Refractions of any Sort of Rays are made, according to certain Ratios of the Sines, and the Ratios of those Sines will be known. But at present, since I know the Refraction of any Ray to be the same, whether it is incident, mixed with heterogeneal Rays, as in the Sun's Light not yet refracted, or is first separated from

Fig. 15.

Fig. 16.

from Heterogeneal Rays: I will ſhew how theſe Proportions may be obtained, by the Refraction of the Sun's immediate Light, in the firſt Place by determining the Proportions of the Sines of Refraction amongſt themſelves, in Reſpect of the ſame Incidence, and then by Compariſon with the common Sine of Incidence. And becauſe it is eaſy to make a Judgment of the intermediate Sorts of Rays, provided the Refractions of the Extreams were known, it will be ſufficient, if I ſhall compare the moſt refrangible Rays of all with the leaſt Refrangible. Therefore, in *Fig* 21, let A B C be a Glaſs Priſm ſo placed, that the Rays both entering and emerging, may ſuffer, as before, the ſame Degree of Refraction: But let there be choſen a clear Day, and the Room be very dark, that the Colours, even to the Extremities of the Spaces they poſſeſs, may be ſeen ſufficiently diſtinct. Then, at the Diſtance of Twenty Feet or more from the Priſm, let the Rays be received on a Paper directly oppoſed to them, and let the Length and Breadth of the Space, (as P T) illuminated

Fig. 21.

nated by the Colours, be meaſured. So a Priſm being uſed, whoſe vertical Angle A C B was 63 Deg. 12 Min. and the Breadth of the Hole admitting the Rays being $\frac{1}{4}$ of an Inch, at the Diſtance X P or X T Twenty two Feet, I found the greateſt Length of the Image P T to be $13\frac{1}{4}$ Inch. nearly, and the Breadth $\frac{2\,5}{8}$ Inch. Now, if the Breadth of this Image be taken from its Length, there will remain $10\frac{5}{8}$ Inch. the Length it ought to have, if the Diſc of the Sun, and the Diameter of the Hole F was infinitely ſmall, that is, if all the Rays had flowed in the ſame right Line O F. That Line therefore of $10\frac{5}{8}$ Inch. ſubtends an Angle, which two Rays incident alike, make by the Inequality of Refraction, whereof one is refracted the moſt of all, having the like Incidence, and the other the leaſt of all: Which Angle therefore, by Calculation, is found to be 2 Deg. 18 Min. But ſince that Angle is made by a double Refraction at X and Y; and beſides ſince both are ſuppoſed equal, a Calculation ſufficiently exact for this Buſineſs might have been per-

for-

formed from one Refraction only, as that made at the Side B C. For if the vertical Angle A C B be bisected by the Plane D C, and the other Half of the Prism D C B, or D C A be conceived to be taken away, the Refraction made at the other Half by the Rays O F obliquely falling on A C, and emerging perpendicularly from the Side D C, or falling perpendicularly on the Side D C, according to one certain Line X Y, and emerging obliquely from the Side B C; the Refraction, I say, thus made at the other Half, would be Half the Refraction at the whole Prism, provided one particular Sort of the mean refrangible Rays were only regarded. Moreover, if all the other Sorts of Rays were at the same Time regarded, that Assertion, though it is not absolutely true, yet it approaches the Truth so nearly, that as to Sense and mechanical Calculation, it may be looked upon as true. Wherefore, since a Geometrical Calculation of both the Refractions made at X and Y is tedious to perform, I shall not fear to do it by a Way more accommodated to Pra-

ctice, though a mechanical one; trusting I ought not to be blamed, if whilst I apply Computations to physical Matters, I shall omit such minute Circumstances, as would occasion a troublesome Work to no Purpose: I shall therefore consider the Refraction from one Side of the Prism only, and because all Rays, besides the mean refrangible ones, ought to be twice refracted by the Half A C D, and once only from the other Half D C B, entering perpendicularly the Plane Side D C, according to the Line X Y: Therefore let the Calculation be made in the Half D C B, that is, at the plane Side B C, it being supposed, that all the Rays flowing according to the same Line X Y, the Angle, which the most refrangible Rays would make with the least refrangible, after they were refracted by the Side B C, would be Half the Angle P Y T, that is, 1 Deg. 9 Min. Now, since the Angle of Incidence of the Ray X Y, from what has been shewn, is 31 Deg. 36 Min. and the Angle of the mean Refraction 54 Deg. 10 Min. let all these be trans-

Fig. 22. ferred to *Scheme* 22, by making C B the Surface terminating the Medium of Glaſs towards A, and that of Air towards F, and the Angle of Incidence XYH to be 31 Deg. 36 Min. and the Angle of Refraction R Y F will be 54 Deg. 10 Min. and the Angle P Y T 1 Deg. 9 Min. *viz.* the Difference of Refraction between the moſt refrangible Rays Y P and the leaſt refrangible Y T: which Angle, let be biſected by the mean refracted Ray Y R, poſſeſſing the Confine of Blue and Green. And thence the Angle PYR, or R Y T will be 34½ Min. Half the whole P Y T. And therefore the Angle P Y E will be 54 Deg. 44½ Min. and the Angle TYE 53 Deg. 35½ Min. and their Sines PG and F T will be 81656 and 80481, whoſe Proportion being reduced to more ſimple Numbers, will be TF to PG, as 69½ to 68½ nearly. When I had often after this Manner made Experiments and Calculations, the Proportions of theſe Sines always came out between the Limits 67 to 66, and 72 to 71; but I ofteneſt fell upon the Proportions *69* to *68*,

68, $69\frac{1}{2}$ to $68\frac{1}{2}$ and 70 to 69, whoſe Difference is ſo ſmall, that it matters not, which is made Uſe of.

THE Ratio of the Sines of Refraction for the extreme Sorts of Rays alike incident being thus found, their Computation to a Sine of Incidence is alſo known: *viz.* which has been before found to be 52400; and by comparing this 52400 to the Sines 81656 and 80481, their Ratio in leſs Numbers will be found $44\frac{1}{2}$ to $69\frac{1}{2}$ and $68\frac{1}{2}$, or $44\frac{1}{4}$ to 69 and 68 nearly; the Refractions being made out of Glaſs into Air. 38. The Sines of theſe Refractions are compared with a common Sine of Incidence.

BUT if on the contrary the Rays are alike incident out of Air into Glaſs, the Proportions of the Sines are without any Trouble diſcovered from what has been already found; as they are reciprocal. Let I be the common Sine of Incidence out of Glaſs into Air, P the Sine of Refraction of the moſt refrangible Rays, R of the mean refrangible ones, and T of the leaſt refrangible. I ſay, that from the reciprocal Proportions of theſe, if $\frac{1}{I}$ be put for the Sine of Incidence 39. The Sines of Rays incident on oppoſite Parts of the ſame refracting Surface are reciprocally proportional.

cidence out of Air into Glaſs, $\frac{1}{P}$ will be the Sine of Refraction of the moſt refrangible Rays, $\frac{1}{R}$ the Sine of Refraction of the mean refrangible Rays, and $\frac{1}{T}$ of the leaſt refrangible. For ſince the Sine of ~~Incidence~~ of the moſt refrangible Ray ~~out~~ of Glaſs into Air is I, and the Sine of Refraction P, the Sine of Incidence of that Ray paſſing backwards by the ſame Lines out of Air into Glaſs will be P, and the Sine of Refraction I; for now that is the Incident Ray, which before was the refracted. The Sine therefore of Incidence of a moſt refrangible Ray out of Air into Glaſs, however incident, is to the Sine of Refraction, as P to I, that is, (by applying the Ratios to P) as 1 to $\frac{I}{P}$, that is, (by applying laſtly to I) as $\frac{1}{I}$ to $\frac{1}{P}$, and by a like Argument it will appear, that the like Sines of a mean refrangible Ray are, as $\frac{1}{I}$ to $\frac{1}{R}$, and the Sines of the leaſt refrangible as $\frac{1}{I}$ to $\frac{1}{T}$. It is manifeſt therefore, that $\frac{1}{I}$ being

ing put for the common Sine of Incidence $\frac{1}{P}$, $\frac{1}{R}$ and $\frac{1}{T}$ will be the respective Sines of each Sort.

I will illustrate it in Numbers. Since $44\frac{1}{4}$ to *69* and *68* is the Ratio of the common Sine of Incidence to the Sines of the greatest disagreeing Refractions out of Glass into Air, it will be as $\frac{1}{44\frac{1}{4}}$ to $\frac{1}{69}$ and $\frac{1}{68}$ or $\frac{69 \times 68}{44\frac{1}{4}}$ (=106 nearly) to *68* and *69*. That is, for the least refrangible Rays as 106 to *69*. These being thus determined, the Proportions of the Sines for the intermediate Rays are easily determined from knowing the Distances of the Colours, which they observe in the coloured Image. So the Rays which approach a little nearer to the Blue than the Yellow, since they fall on the middle of the Image, they will have the intermediate Proportion of the Sines $44\frac{1}{4}$ to $68\frac{1}{2}$ or 106 to $68\frac{1}{2}$ nearly; and so of others.

40. The Refraction of Glass is illustrated.

41. From the Refractions of the extreme Sorts, it is easy to make a Conjecture of the intermediate ones.

42. It is shewn by a *Theorem*, how from the Refractions of heterogeneal Rays at Glass or any other Medium being determined amongst themselves, the Refractions at any other Mediums contiguous to the Air may also be determined amongst themselves without the new Trouble of Experiments. *Fig.* 23.

AFTER the same manner that the Refractions at Glass are determined, it may be done for other Mediums, but it will be worth while now to shew, how the Measures of those Refractions may be determined for any other proposed Medium from their Sines being thus determined for Glass; and that although it be contiguous to any other Medium than Glass. In *Fig.* 23. Let A B be a Surface terminating the Air on the Side F, and Glass on the Side G, at any Point whereof X let be drawn the Line F X G perpendicular to it, and besides let be conceived the right Line I X to be drawn in an Angle I X A infinitely small, according to which let all the Rays of all Forms be supposed to be incident, and in X to be refracted: As the most refrangible ones towards P, the mean refrangible towards R, and the least refrangible towards T, and the other intermediate ones towards intermediate Parts: Moreover let be drawn any Line G H parallel to the Line of Incidence I X, that is, perpendicular to F G, but let it cut the Rays in the Points

Points P, R and T, from which let fall P C, R D and T E perpendicular to the refracting Surface A B. These Things being thus determined and described for Glass, if any Medium be now conceived to be substituted in the room of that of Glass, all things remaining the same, and a refracted Ray X *r*, belonging to any mean refrangible Ray incident in the Line I X at X, be drawn cutting the right Line D R in *r*. Which I suppose to be done, because I have before shewn, how the Refractions of the mean refrangible Rays may be investigated for any Mediums. Then through the Point *r* let be drawn the right Line *r t* cutting the Lines C P and E T in *p* and *t* perpendicularly, and let *p* X and *t* X be joined. I say, that the most refrangible Rays incident in the said Line I X will be refracted into the Line X *p*, and the least refrangible into the Line X *t*, and the Rays of any Sort, which Glass refracted to any Point of P T, will be refracted to a correspondent Point of the right Line *p t* by the other Medium, that is supposed to be substituted for Glass; those

Points of the Lines P T and *p t* being reckoned correſpondent, thro' which any right Line parallel to D R paſſes. The manner therefore appears, how the Refractions of any Rays incident with the greateſt Obliquity out of Air into any propoſed Medium may be determined, the Refraction of one Sort of Rays only into the Medium being known; and the Proportions of the Sines from that moſt oblique Refraction being determined, the Refractions of the ſame Rays will be given to any other given Incidence.

43. Of the Certainty of the *Theorem.* THE Certainty indeed of this *Theorem* I have not yet derived from Experiments, but ſince it ſeems ſcarcely to differ much from the Truth, I have not doubted for the preſent to aſſume it *Gratis*. Hereafter perhaps, I ſhall either confirm it by Experience, or correct it, if I ſhall find it to be falſe.

44. Of its Calculation. As to Calculation, it may be eaſily made from this Proportionality, that the Sine of Incidence of the Ray I X (that is, the Sine of 90 deg.) is to the

Sine of Refraction (suppose that made in the Line X R) as X R to R G; so in Glass it will be X R. R G :: 106. $68\frac{1}{2}$, and X P, P G :: 106. 68, and X T. T G :: 106. 69; and thence it will be deduced, that G P. G R. G T :: 39. $39\frac{1}{2}$. 40. Which Proportions being once found they may be reserved, to the End that by their means Refractions to other Mediums than Glass may be determined. For any Medium being proposed, let be taken X E = 40, D E = $\frac{1}{2}$ and C D = $\frac{1}{2}$, and let the Perpendiculars C P, D R and E T be erected. Then from the given Proportion of the Sines of Refraction of the mean refrangible Rays, that is, from the given Proportion of X *r* to X D, there will be given the Point *r* and the Length D *r*, to which C *p* and E *t* are equal: And the Points *p* and *t* being thus given, there are given the Ratios of X *p* and X C, that is, of the Sines of Incidence and Refraction for the most refrangible Rays, as also the Ratios of X *t* and X E, that is, of the Sines of Incidence and Refraction for the least refrangible Rays. So for a Surface bounding Water and

Air, those Sines are as 68 to 90 for the least refrangible, and as 68 to 91 nearly for the most refrangible Rays. The Proportions of the Lines X C, X D and X E being thus found, the Measure of the Refractions out of Air into any proposed Medium, and made at any Incidence, may be determined by another not inelegant *Theorem.*

45. Another *Theorem* to perform the same Thing. *Fig.* 24.

IN the Line F X (*Fig.* 24.) perpendicular to the refracting Plane A B let be taken any Point F, which let be supposed lucid, and let any Line F *d* be drawn cutting A B in *d*, and let it be conceived to be a mean refrangible Ray, whose refracted Ray out of Air into the proposed Medium let be *d* M, which produced backwards let it cut F X in *f*. Moreover let be made F *d*. F *e* : : X D. X E (: : $39\frac{1}{2}$. 40.) and F *d*. F *c* : : X D. X C (: : $39\frac{1}{2}$. 39.) and with the Center F and Intervals F c and F *c* let be described Circles cutting A B in *e* and *c*, and let be joined F *e*, F *c*, *f e*, *f c*, and let *f e* and *f c* be indefinitely produced towards N and L. I now say, if the least refran-

Air, thoſe Sines are as 68 to 90 for the leaſt refrangible, and as 68 to 91 nearly for the moſt refrangible Rays. The Proportions of the Lines X C, X D and X E being thus found, the Meaſure of the Refractions out of Air into any propoſed Medium, and made at any Incidence, may be determined by another not inelegant *Theorem*.

45. Another *Theorem* to perform the ſame Thing. *Fig.* 24.

IN the Line F X (*Fig.* 24.) perpendicular to the refracting Plane A B let be taken any Point F, which let be ſuppoſed lucid, and let any Line F *d* be drawn cutting A B in *d*, and let it be conceived to be a mean refrangible Ray, whoſe refracted Ray out of Air into the propoſed Medium let be *d* M, which produced backwards let it cut F X in *f*. Moreover let be made F *d*. F *e* : : X D. X E (: : $39\frac{1}{2}$. 40.) and F *d*. F *c* : : X D. X C (: : $39\frac{1}{2}$. 39.) and with the Center F and Intervals F c and F *c* let be deſcribed Circles cutting A B in *e* and *c*, and let be joined F *e*, F *c*, *f e*, *f c*, and let *f e* and *f c* be indefinitely produced towards N and L. I now ſay, if the leaſt refran-

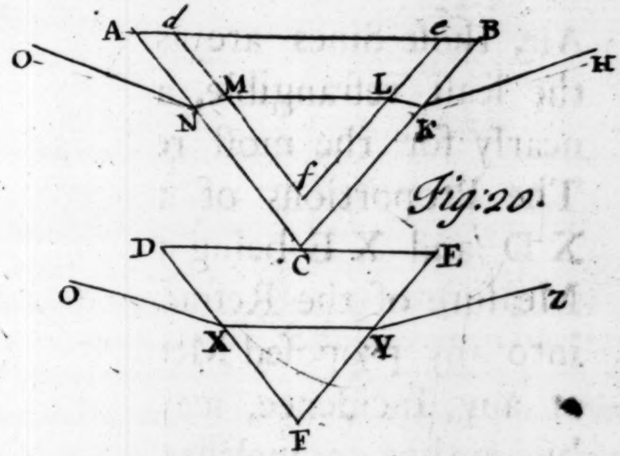


Fig: 20.

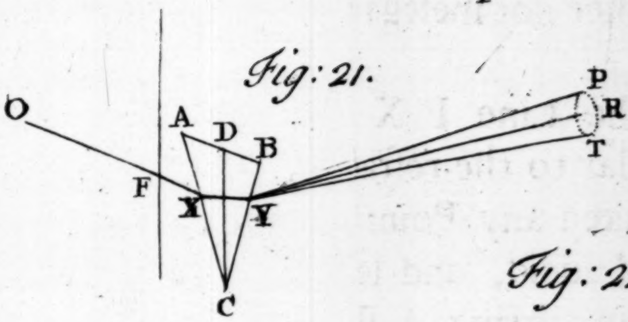
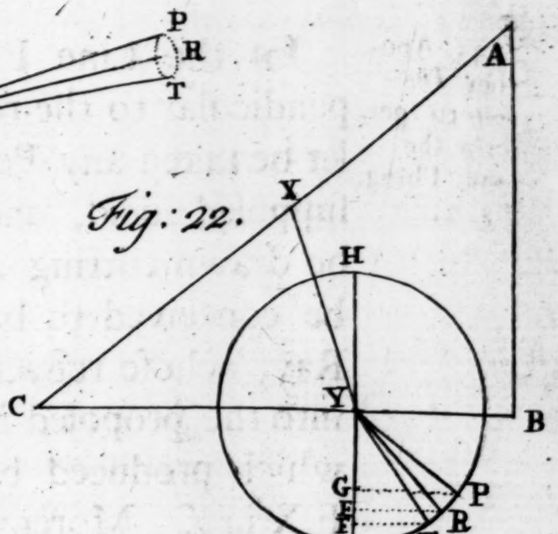


Fig: 21.



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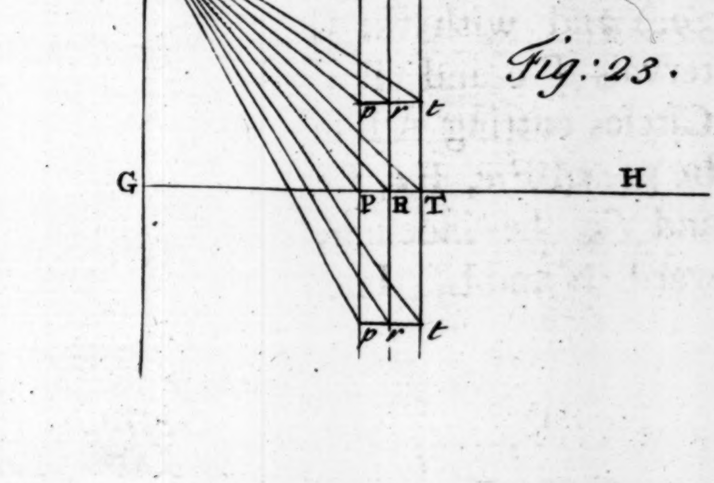


Fig: 23.

refrangible Ray falls in the Line F *e*, that it will be refracted into the Line *e* N; and if the most refrangible Ray falls in F *c*, that it will be refracted into *c* L, and so the Rays of any Intermediate Sorts, flowing from the Point F, and falling upon their corresponding Points between *c* and *e*, will be so refracted by the proposed Medium, as if they had all flowed from the Point *f*; those Points between C and E, and *c* and *e* being looked upon as corresponding, whose Distances from X and F respectively are in the same Ratio with D X to *d* F. For the demonstrating which *Theorem*, the Two following *Lemmas* are premised.

1. Two Points *c*, *d* being taken (*Fig.* 24.) in any Line A B, and other two Points *f* and F in its perpendicular F X, and *f d*, F *d*, *fc* and F *c* being joined, the Difference of the Squares of the two Lines *fd* and F *d* meeting in *d*, will be equal to the Difference of the Squares of the other two *fc* and F *c* meeting in *c*. For since *f d* q = *f* X q + X *d* q, and F *d* q = F X q + X *d* q, the

46. For demonstrating this *Theorem*. *Lemma* 1. *Fig.* 24.

the Difference $fdq - Fdq$ will be $= fXq - FXq$; and for the ſame Reaſon the Difference $fcq - Fcq$ will be $= fXq - FXq$. Wherefore the ſaid Differences thus equal to the ſame Third, are equal amongſt themſelves. Q. E. D.

47. *Lemma* 2. *Fig.* 25. 2. If any Ray F G (*Fig.* 25.) falls on the Surface A B, and is refracted towards H, the Line G H being drawn backwards, that it may cut the perpendicular F X in f; I ſay, that $fG . FG ::$ Sine of Incidence to the Sine of Refraction: And on the contrary, if $fG . FG ::$ Sine of Incidence to the Sine of Refraction, then fGH will be the Refracted Ray of F G. For let be taken $fK = FG$, and let fall K L perpendicular to F X, which being done, ſince the Angle G F X is equal to the Angle of Incidence, and the Angle GfX to the Angle of Refraction, having Reſpect to a Circle, whoſe Semidiameter let be F G or fK; but $fG . fK :: GX . KL$. that is $fG . FG :: GX . KL$. Q. E. D.

THESE

THESE Things being premised, the *Theorem* is thus demonstrated. In *Fig*. 24. Let be drawn I X the most oblique Line, according to which Rays of all Forms are supposed to be incident, out of Air at X, and to be refracted into the proposed Medium, the most refrangible ones towards *p*, and the least refrangible towards *t*, and let these be cut by Lines erected perpendicular at the Points D, C and E in the Points *r*, *p* and *t*, as was explained at *Fig*. 23. Now, since the Sines of Incidence and Refraction of these Rays are appointed to be as X *r* to X D, X *p* to X C and X *t* to X E respectively, if besides it shall be demonstrated, that *f d* to F *d*, *f c* to F *c* and *f e* to F *e* are respectively in the same Ratio; that is, that *f d*. F *d* :: X *r*. X D :: Sine of Incidence to the Sine of Refraction of the mean refrangible Rays, and *f c*. F *c* :: X *p*. X C :: Sine of Incidence to the Sine of Refraction of the most refrangible Rays, the *Proposition* will be manifest by the second *Lemma*. And as to the mean re-

48. The Demonstration. *Fig*. 24.

refrangible Rays, ſince fd is ſuppoſed the refracted Ray of F d, it will be (by the ſecond *Lemma*) fd to F d as the Sine of Incidence to the Sine of Refraction, that is, as X r to X D; But it is now propoſed to demonſtrate the ſame Proportionality in the other Sorts of Rays, as that it is fc. F c :: X p. X C. For it is F c F d :: X C. X D; as alſo F d. fd :: X D. X r by Hypotheſis. Wherefore by Permutation and Converſion it is F c. X C :: F d. X D :: fd. X r. and by ſquaring F c q. X C q :: F d q. X D q :: fd q. X r q. and by diminiſhing by the Terms of the equal Ratio, F c q. X C q :: fc q. C p q + X C q, (X p q). *Laſtly*, By extracting the Roots of the Terms, and by Permutation it is fc. F c :: X p. X C. Wherefore fc or c L is the refracted Ray of F c by the ſecond *Lemma*. Q. E. D. And by the ſame Argument it will appear, that e N is the refracted Ray of F e. And the ſame is to be underſtood of other Rays variouſly poſſeſſing the intermediate Spaces according to the various Degrees of Refrangibility.

OF

OF the Measuring the Refractions of Surfaces contiguous to the Air, we have said enough; but if the same Thing be required to be done for other Surfaces contiguous to the Air on no Part, let (in *Fig.* 26.) A B *b* H, and *a b n m* be any two Mediums contiguous in the Plane Surface H *b*, and incompassed with Air. And let the Plane A B be parallel to H *b*, and in it let be taken the Point X, to which let be drawn X V perpendicular, and I X the most oblique Line in which (as before) let the Rays of all Forms be incident, and according to the Degree of Refrangibility be refracted to P, R and T, and other intermediate Places. The Refractions of these Rays thus incident on the proposed Surface *a b* are now to be sought, and since the Refractions of the mean refrangible Rays to any Surfaces were before exposed, let of the Ray X R the refracted one be R M, and let it be drawn backwards, till it cuts the Perpendicular X V in *f*, and let moreover be drawn *f* P, *f* T, and let them be produced to L and N. I say, that P L will

49. The Refractions of heterogeneal Rays, from Surfaces contiguous on no Side to the Air, are also determined by a *Theorem.*

Fig. 26.

will be the refracted Ray of X P, and T N of X T, and all the Rays of the other Forms falling between P and T, will be ſo refracted, as afterwards to diverge from the Point *f*. For let it be conceived, that the Medium *a b n m* be produced out further towards *a m*, than the Medium A B *b* H, ſo that the Part of its Plane *a* H *b* between H and *a*, may be contiguous to the Air, and to ſome Point in it F let be drawn the perpendicular F *g*, alſo the moſt oblique Line *j* F, in which let the Rays of all Forms be incident, and according to their Degree of Refrangibility be refracted to *p*, *r*, *t*, and the intermediate Places, juſt as it was done at the Surface A B of the other Medium. Beſides, let be taken F D = G R, and be drawn D *r* parallel to F *g*, that it may cut the Ray F *r* in *r*, whence let fall *r g* perpendicular to F *g*, and cutting the other Rays F *p* and F *t* in *p* and *t*. Now, Since it is *g r* = G R, it will alſo be *g p* = G P, and *g t* = G T, from what was ſhewn at *Fig.* 23. And moreover, from what was ſhewn at *Fig.* 18. ſince the Refraction of the

Rays

Rays incident in the parallel Lines I X and *j* F on the Medium *abnm*, is the ſame, whether they immediately enter out of the Air, as is done at F, or do firſt pervade another Medium, as A B *b* H terminated by parallel Planes; it follows, that the Rays refracted after either of theſe Ways into the ſaid Medium *a b n m*, are parallel to the homogeneal Rays refracted after the other Way into the ſame Medium, that is, that F *p* is parallel to P L, F *r* to R M, and F *t* to T N. Wherefore if the refracted Rays P L, R M and T N be drawn backwards, till each meet the perpendicular G X, they will conſtitute with it, and the Baſes G P, G R and G T Triangles, ſimilar to the Triangles *gp* F, *gr* F and *gt* F, and alſo equal to them; for their Baſes *gp* and GP, *gr* and G R, *gt* and G T are reſpectively equal to one another. Wherefore, ſince the Vertex's of theſe Triangles meet at the ſame Point F, the Vertex's of thoſe Triangles ſhall meet at ſome other Point *f*. That is, the Rays P L, R M and T N, the refracted Rays of X P,

XR and XT, ſhall all diverge from the ſame Point *f*. Q. E. D.

50. The *Theorem* is promoted by ſome *Notes*. THIS being ſhewn, the following Things offer themſelves to our Obſervation. 1. That the Proportions of the Sines of Incidence and Refraction, made at the Surface H *b*, are eaſily determined from theſe. For as to the moſt refrangible Rays *f* T is to X T, &c.

2. HENCE if the Proportions of the Sines of Refraction out of Air into any two propoſed Mediums, at like Incidences, be given; the Proportions of the Sines of Refraction out of one o the Mediums into the other will be eaſily given, *viz.* by dividing the Sines of the latter Medium by the correſponding Sines of the anterior Medium. So when the Refraction is made out of Air into Glaſs, the ſaid Sines are as *68*, *68½*, *69*; and when it is made out of Air into Water, they are as *90*, *90½*, *91*. Therefore, when it is made out of Water into Glaſs, they will

will be as $\frac{68}{90}$, $\frac{68\frac{1}{2}}{90\frac{1}{2}}$, $\frac{69}{91}$, that is, as 281, 281½, 282 nearly.

3. If any third Medium denſer than Air is placed behind the Medium *a b n m*, touching it in the Surface *m n*, that is conceived to be a Plane, and parallel to A B and *a b*, and if Rays diverging from the Point *f* (as it was juſt now ſhewn) fall upon it at the Points L, M and N; after they are refracted in the ſame, they will diverge again from any other Point *x*, that is ſituated in the Perpendicular X G, and ſo on *in infinitum*, however many the Mediums be, that are ſeparated from one another by parallel Planes, and follow one another in Order. But if Air immediately ſucceeds the Medium *abnm*, that Point *x*, from whence the emerging Rays tend, will be ſituated at V in the very refracting Surface, becauſe they will emerge parallel to the greateſt oblique Line I X, in which they were at firſt incident out of the Air; if they may be ſaid to emerge, which

never

never divaricated from the refracting Surface.

Fig. 24. 4. IF the Rays, diverging from any Point F situated in the Air, tend to the Points *c*, *d*, *e*, after the manner, that I explained at the 24 *Scheme*, and then pass through various refracting Planes parallel to A B, they will every one always diverge from some one Point, that is situated in the perpendicular of the Plane, that passes through F, not otherwise than if they had been incident on the Plane A B, proceeding in the most oblique Line I X, and the Lengths of the Rays intercepted between the refracting Points and the said Perpendicular, are as the Sines of Incidence and Refraction at every Plane, that they respect. The Demonstrations of which Assertions, since they are easily derived from what has been said before, I omit, that I might not seem to dwell too much on this Affair.

Section III.

Of the Refractions *of* PLANES.

THE Laws of Refractions having been laid down, other Affections of Rays tranſmitted through different Mediums are now to be delivered; and in the firſt Place I will deſcribe the Refractions of Planes for the Sake of the Doctrine of Colours hereafter to be explained. Then I ſhall declare the Properties of ſpherical and other Surfaces, both that the Phænomena of Colours thence ariſing may be detected, and alſo that the Conſtruction of the Inſtruments ſerving to optical Uſes may be the more truly known. But firſt I ſhall conſider the Refractions of a ſingle Plane, then the repeated Refractions of Planes.

Of the REFRACTIONS *of a ſingle* PLANE.

As to what relates to Rays of any the ſame Sort, their Affections (upon theſe Principles, that the Rays of Light in a ſimilar Medium are direct; that their Refraction is made in a Surface perpendicular to the Surface of the refracting Medium, and that the Sines of Incidence are conſtantly proportional to the Sines of the Refractions made in another ſimilar Medium) are delivered in Dr. *Barrow's Lectures;* and therefore it will be ſufficient to enumerate here ſome of them under the Form of *Lemmatical Propoſitions* without their Demonſtrations.

PROP. I.

[a] *The incident Ray of any refracted one becomes interchangeably the refracted Ray of an incident one.*

[a] *Barrow's Optical Lectures, Lect.* III. *Art.* III.

PROP.

PROP. II.

[b] *To an equal or a greater Angle of Incidence, there belong an equal and a greater both Angle of Refraction and refracted Angle, and the contrary.*

PROP. III.

[c] *Of Incident Rays to exhibit their refracted ones.*

TAKE an Inſtance in Rays diverging out of a rarer Medium into a denſer one.

IN *Fig.* 27, let F be a Point emitting the Rays F R, F *r* and innumerable others towards the refracting Surface A R, and let F A be the perpendicular Ray, which produce to K, that it may be A F to A K, as the Sine of Refraction to the Sine of Incidence, and at K erect the perpendicular K L. Which being done, produce backwards *Fig.* 27.

[b] Barrow's *Optical Lect. Lect.* III. *Art.* IV. & VI.
[c] *Ibid. Lect.* IV. *Art.* V.

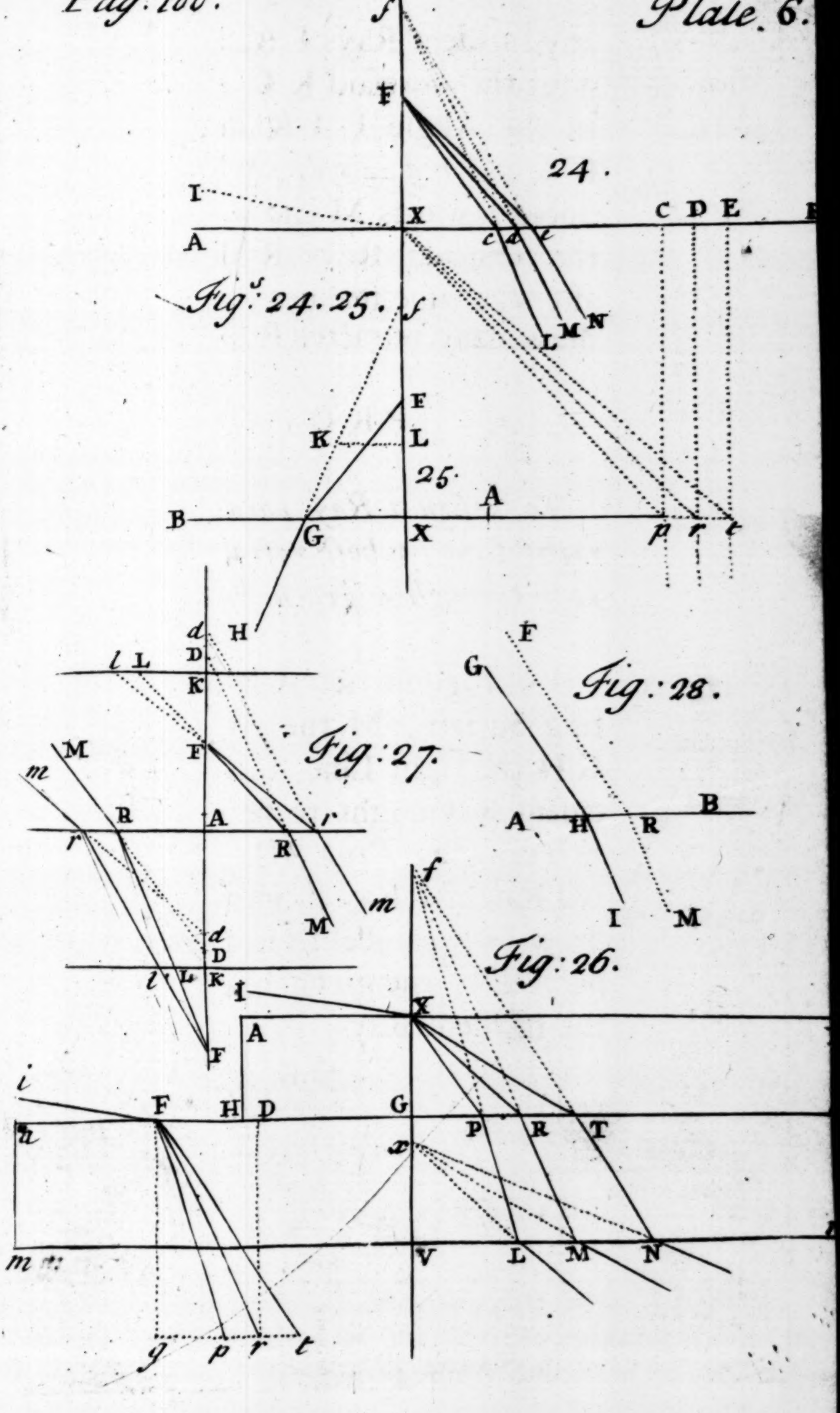
any incident Rays F R, F *r*, until they meet the aforesaid K L in L and *l*, and in the Angle F A R inscribe R D = R L and *r d* = *r l*. Which being produced towards M and *m*, you will have the refracted Rays R M and *r m*; and after the same manner you may speedily draw many refracted Rays.

PROP. IV.

To design a Ray parallel to a given right Line, whose refracted Ray shall pass through a given Point.

Fig. 28. IN *Fig.* 28. let A B be the refracting Surface, M the given Point, and G H the right Line, to which the incident Ray ought to be parallel. And first draw by *Prop.* III. H I the refracted Ray of a Ray incident in G H, and draw M R parallel to it, and F R drawn parallel to the given Line G H will be the incident Ray.

Pag: 100. Plate. 6.



Figs: 24. 25.

Fig: 27.

Fig: 28.

Fig: 26.

PROP. V.

To design a Ray proceeding from a given Point, whose refracted Ray shall become parallel to a right Line given in Position.

IT is performed after the manner of the IVth *Proposition*, the Denomination of the Rays being changed according to *Prop.* I.

PROP. VI.

To design a Ray proceeding from a given Point F, *whose refracted Ray shall pass through another given Point* M.

THROUGH F and M (*Fig.* 29.) let be drawn Perpendiculars to the refracting Surface, and (the Ray falling upon a denser Medium) let it be made A E to A F as the Sine of Incidence to the Root of the Difference of the Squares of the Sines of Incidence and Refraction; also T to M I as the Sine of *Fig.* 29.

Refraction to the same Root. And in the Angle A I M let be inscribed the right Line R H passing through E and equal to T, and let be joined F R, R M; For F R, R M will be the Rays sought.

WHEN the Ray is incident upon a rarer Medium, the Denomination being (acording to *Prop.* I.) changed, it is solved, as before.

BUT how in a right Angle is to be inscribed a given right Line, which shall pass through a given Point, is shewn in Dr. *Barrow's* Vth *Lecture* by the Intersection of an Hyperbola and a Circle.[a]

PROP. VII.

[b] *Of Rays, that diverge, are parallel or converge at a plane Surface, their refracted Rays in like manner will*

[a] *Viz.* in *Lect. Optic. Lect.* V. *Art.* VII. But this was done afterwards by the Dr. more generally, as well as more elegantly, at the Beginning of his VI. *Geometrical Lecture.*

[b] Barrow's *Optical Lectures, Lect.* IV. *Art.* II. &c.

diverge,

diverge, be parallel or converge. And on the contrary.

PROP. VIII.

To find the Point from which these refracted Rays diverge; or to which they converge.

CAS. 1. WHEN the Inclination of the Rays is defined, draw the refracted ones by *Prop.* III, IV, V, or VI, and you will have the Intersection.

CAS. 2. BUT when the Inclination is indefinitely less than any given, the *Problem* comes to the same Thing, as if you sought a Point in an oblique refracted Ray, which distinguishes and falls between the Intersections of the Rays lying on both Sides, and which ought to be looked upon as the Center of Radiation, or the Place of the Image in respect of the Eye thro' the Center of whose Pupil that Ray is to pass. But its Invention is thus. In *Fig.* 30. let D R M be a refracted Ray of any incident one F R N, and let *Fig.* 30.

let F be the Center of Radiation of the incident Rays whether diverging or converging, and let F A insisting perpendicular on the refracting Plane cut R M in D. Now from A let fall to those Rays the Perpendiculars A G and A H, and make R F. R *f* : : F G. D H, and *f* will be the Center of Radiation of R M, and of the other refracted Rays near R M lying on each Side. [c]

SCHOL. BUT this *f* is the Concourse only of the Rays lying in the Plane F A R, the refracted Rays of the others lying out of the Plane F A R will cut the Ray R *f* neither in the Point *f*, nor any where else at all, if you except those only, whose incident Rays lie in a conical Surface, whose Axis is A F, Vertex F, and Semi-angle A F R, in as much as they will all cut the said R *f* in the Point D, which is placed in the Axis F A. And so the Centers of Radiation of this R *f* are chiefly two, one *f* made by the refracted Rays of those lying in the Plane F A R, and the other by the

[c] *See* Barrow's *Optical Lectures, Lect.* V. *Art.* XV. *&c.*

refract-

refracted Rays of those lying in the conical Surfaces described with the Axis F A, and Angles A F R, A D R. But as to the rest of the Rays placed otherwise every where about F R, their refracted Rays most nearly approach the Ray R *f* somewhere between D and *f*. So that in respect of the Eye, through the Center of whose Pupil the Ray R M passes, the Place of the Image ought to be diffused through the whole Space *f* D; or rather, since the Space *f* D is the Image of one Point only F, we ought to fix in it for the sensible Image some one Point, that may possess the very middle of all the Light proceeding from it towards the Eye, lying between the Points D and *f* nearly in the middle Distance. But the accurate Determination of that Point, when regard is to be had to all the Rays refracted from F towards the Pupil of the Eye, affords a *Problem* very difficult to be solved, unless the Assertion be founded on a certain Hypothesis at least probable, if not accurately true. As since as equal a Number of Rays seems to flow towards the Eye from the Limit D and

D and other neighbouring Points, as from the Limit f and other Points alike near to it; the Place of the Image ought so to be fixed in the middle of these Limits, that the Angle, which two Rays from D and f converging to any the same Point of the Pupil do include, may be always very nearly bisected by a Ray proceeding from the Place of Vision, to the same Point of the Pupil. Which Hypothesis being admitted, there is nothing else to be done, than that it be made $Mf + MD. MD :: fD. DZ$, and Z will be the sought Place of Vision of the Point F, it being supposed that at M is placed the Eye; for since it was put $Mf + MD. MD :: fD. DZ$; it will be by Division, $Mf. MD :: fZ. DZ$. And therefore three Lines being drawn from f, D and Z to M or rather to some Point indefinitely near to this M, the Angle, which the two external Lines contain, will (by 3, 6. *Elem.*) be always very nearly bisected by the interjacent Line.

THESE

THESE few Things about homogeneal Rays being curſorily remarked for the Sake of what follows; in order to get a fuller Knowledge herein, I adviſe the Peruſal of the *Lectures*, which the Reverend Dr. *Barrow* has more largely compoſed on the ſame Subject; and I proceed immediately to diſcourſe of heterogeneal, or unequally refracted Rays.

PROP. IX.

Of Rays of different Sorts, flowing from a lucid Point, whoſe incident ones are neareſt to one another; thoſe alone can be refracted to a Focus or other common Point, which lie in a Plane paſſing through both Points, and perpendicular to the refracting Plane.

FOR the Refraction of any Ray whatever is always made in a Plane perpendicular to the Surface of the refracting Medium, and two ſuch Planes cannot paſs through both Points.

PROP.

PROP. X.

Of Rays of different Sorts flowing from a given Point, whose refracted ones converge to another given Point, they, that are most refrangible, do most divaricate from the right Line lying between the Points of their Concourse or of their Radiations.

Fig. 31. LET F P*f*, F Q*f* (*Fig.* 31.) be dissimilar Rays meeting on this and that Side in F and *f*, and it is manifest, that they will not entirely coincide; for so the Refraction would be equal contrary to Hypothesis. Nor can the greater refrangible Ray be nearer to the right Line F *f*. For so on Account of the greater Obliquity from the Part of the denser Medium, its Refraction would be greater by *Prop.* II. and Hypoth. that is, the Angle F *p f* would be less than the Angle F Q *f* contrary to 21. 1 *Elem.* It remains therefore, that F P *f* is more refrangible, that divaricates more from the right Line F *f*.

LEM-

LEMMA I.

Four Lines GB, GC, GD, GE (*Fig.* 32.) *being so drawn from a given Point* G *to a given right Line* EB, *that it be* GB. GC :: GD. GE. *The Angle* BGC, *which the least* GB *constitutes with either of the intermediate ones as* GC, *is greater than the Angle* DGE *constituted by the other intermediate* GD, *and the greatest* GE. *Fig.* 32.

FOR with the Center G, Radius GE let be described a Circle EK, and let the Radius GK be drawn, constituting the Angle DGK equal to the Angle BGC, and let the Points K, D be joined: And the Triangles GDK, GBC will be similar on Account of the equal Angles at G and the Sides about them proportional, [a] *viz.* GB. GC :: GD. (GE) GK. Wherefore the Angle KDG = Angle CBG, but Angle EDG [b] > angle CBG. Therefore the Line KD > ED [c] and Angle KGD > Angle DGE [d], that is,

[a] 6.6. Elem. et Hyp.
[b] 16. 1. Elem.
[c] 7. 3. Elem.
[d] 21. 1. Elem.

is, Angle C G B > Angle E G D. Q. E. D.

LEMMA II.

These Angles being supposed infinitely small, and G A *let fall perpendicular to the Line* E B, *it will be Angle* E G D. *Angle* C G B :: B A. D A.

For from the Points B and D to the Lines G C, G E let fall the perpendiculars B R and D *r*, and the aforesaid Angles will be to one another as is $\frac{D\,r}{D\,G}$ to $\frac{B\,R}{B\,G}$ *viz.* by putting these Lines B R and D *r* to be equipollent to the infinitely small Arches, by which those Angles are subtended. But it is B G. C G :: D G. E G by Hypoth. and by Division B G. C R :: D G. E *r*. Also on Account of the similar Triangles B A G, C R B, it is B A. A G :: C R. B R, and by a like Reason E A or D A. A G :: E *r*. D *r*, or A G. D A :: D *r*. E *r*. Wherefore by adding equal Ratios it is B A. A G + A G. D A (:: B A. D A) :: C R. R B + D *r*. E *r* (and the Terms of the last Ratios being

ing

ing permuted) :: C R. E r. + D r. B R (and an equipollent Ratio being ſubſtituted for C R. E r) :: BG. DG + D r. B R (and the Terms being applied to one another) :: $\frac{D\,r}{D\,G}$. $\frac{B\,R}{B\,G}$. it is therefore B A. D A.: $\frac{D\,r}{D\,G}$. $\frac{B\,R}{B\,G}$. that is, as the Angle E G D to the Angle C G B. Q. E. D.

PROP. XI.

Heterogeneal Rays being incident, according to the ſame right Line, the more oblique their Incidence is, cæteris paribus, *the greater will be the Difference of the Refraction.*

In *Fig.* 33. Let F G be the Line, according to which two Rays are incident, whereof one the moſt refrangible proceeds towards P, and the leaſt refrangible towards T, and the Angle P G T will be the Difference of Refraction. Alſo F H is an obliquer Line than F G, and according to this let other two like Rays be incident, whereof the moſt refrangible is refracted towards p, and the leaſt refrangible towards t, and in like Manner the Angle *Fig.* 33.

Angle p H t will be their Difference of Refraction. I now say, that the Angle p H t is > P G T. For let fall FA perpendicular to the refracting Plane, which may cut the refracted Rays continued backwards in D, E, L and M, and to this from the Point G let be drawn two Lines G B, G C parallel to H L, H M. Now since the three
No. 25, 26, &c. Lines G F, G D, G E are (from the Nature of Refraction before described) in a given Ratio, and the other Three H F, H L, H M in the same Ratio, H L. H M :: G D. G E will be proportional, but it is H L. H M :: G B. G C, on Account of the similar Triangles L M H and B C G. Wherefore G B. G C :: G D. G E. And consequently the Angle B G C > Angle E D G by *Lem.* I. that is Angle L H M > Angle D G E, or the Angle p H t > Angle P G T. Q. E. D.

But that a fuller Determination may be had of the mutual Proportions (in *Fig.*
Fig. 33. 33.) of the Angles P G T and p H t, I say moreover, that they are amongst one another very nearly, as the Lines A B and A D; *viz.* the Segments of the Bases of equally high Triangles, whereof

of one E G D is constituted by the Rays G P and G T, meeting with the Perpendicular A F, and the other C G B is similar to the Triangle M H L constituted in like Manner by the Rays H *p* and H *t*. For the Angles E G D and C G B, if they were infinitely small, would be to one another as A B to A D by *Lem.* II; but those are by Hypothesis equal to the Angles P G T and *p* H *t*. Wherefore also P G T and *p* H *t*, provided they were infinitely small, would likewise be as A B to A D; and by the like Reason it is plain, that they are very nearly, as A C to A E. *viz.* their Ratio does always lie between these two Ratios, and therefore we shall still approach the nearer to the Truth, by making Use of the intermediate Ratio. *viz.* which is P G T to *p* H *t* as A B + A C to A D + A E, or as $\sqrt{AB \times AC}$ to $\sqrt{AD \times AE}$ nearly.

PROP. XII.

To design Rays of different Sorts, flowing from a given Point, whose re-

fracted Rays shall pass through another given Point.

WHEN one of the Points is infinitely distant, that the Rays on that Side are parallel; the Business is done by the IVth and Vth *Propositions*; and by the VIth *Proposition*, when both are infinitely distant.

SCHOL. IT will be worth While to shew, how from the given Position of any Ray, all the rest are more expeditiously determined.

Fig. 34. CAS. 1. Let in *Fig.* 34. F T, F R, F P be Rays proceeding from F, whose refracted ones T O, R M, P K are to be parallel. And of the Ray F T let the Sine of Incidence be to the Sine of Refraction, as I to T; as of the Rays F R and F P let those Sines be, as I to R and to P. Now, that any of these being given in Position, the rest may be readily designed, let fall F A perpendicular to the refracting Surface, and in the Angle F A T inscribe T E, T D, on this Condition, that it may be


E
F
A

Plate 7th

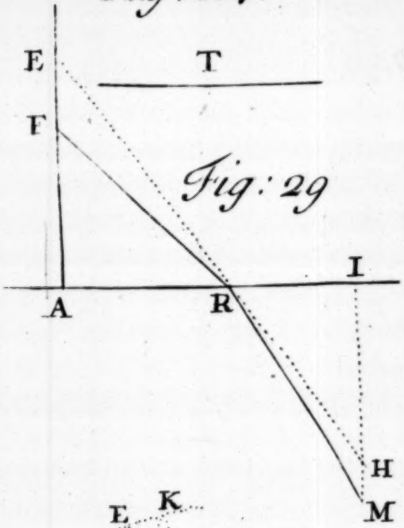


Fig: 29

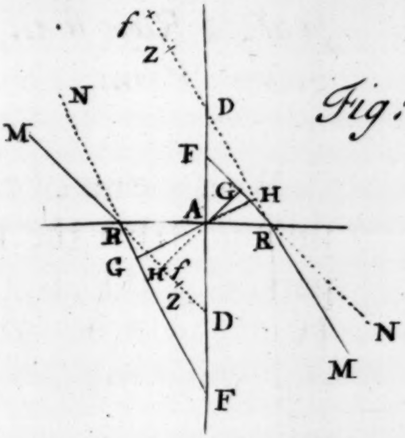


Fig: 30.

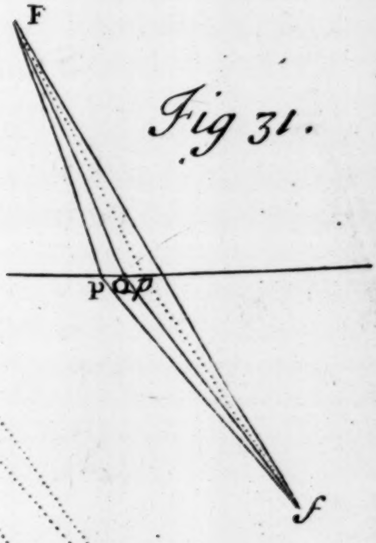


Fig 31.

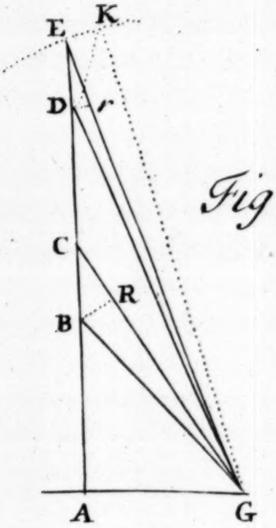


Fig: 32.

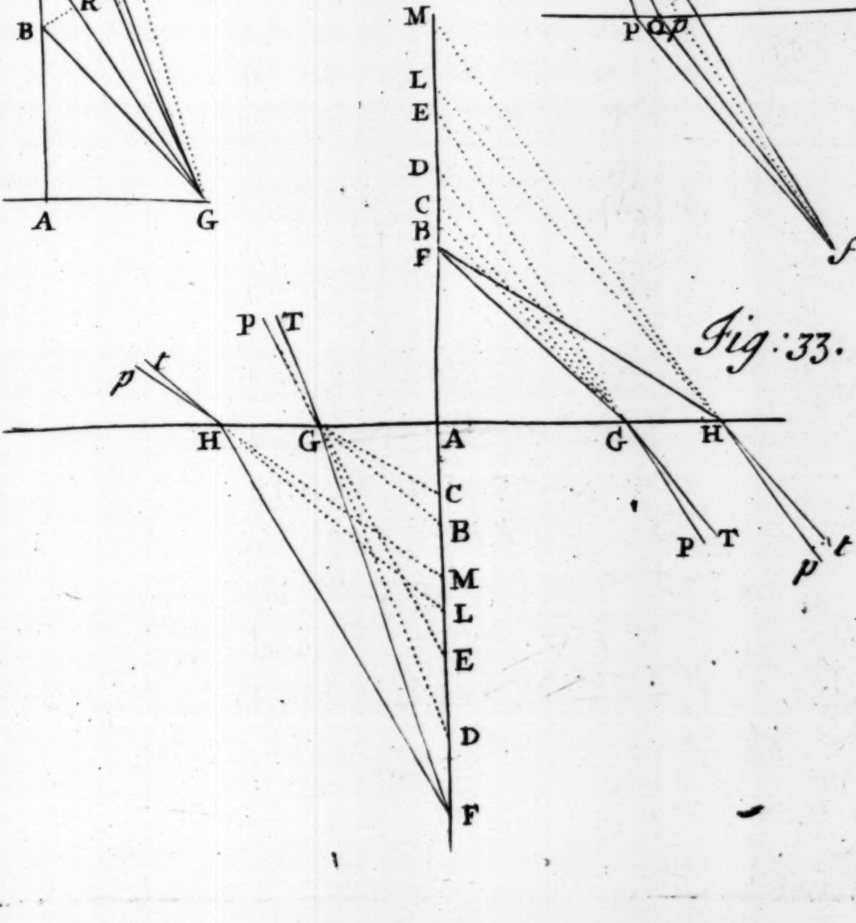


Fig: 33.

T. R, P :: T F. T E. T D, and draw F R, F P parallel to T E, T D. I say it is done. *viz.* the refracted Rays T O, R M meeting with the Perpendicular D A in G and H, it will be I. T :: T G, T F, and besides, since it is T. R :: T F. T E (Hyp.) it will be by Equality I. R :: T G. T E, but it is I. R :: R H. R F; therefore T G. T E :: R H. R F, and consequently Since T E and R F are parallel (by *Hypoth.*) T G and R H will be also parallel. Q. E. D. And there is the same Reason in Relation to the Parallelism of the Ray P K.

CAS. 2. IF the incident Rays being parallel, the refracted Rays converge to a given Point, you may nevertheless perform, what was purposed, as appears from the aforesaid *Prop.*

CAS. 3. *Lastly*, If the incident Rays diverge, and the refracted ones converge, the *Problem* is solid, but is in some Sort reduced to a plane one, by feigning the Difference of Refrangibility to be infinitely small, which since

it is always very little, I ſhall exhibit not unwillingly a Solution on that Hypotheſis.

Fig. 35. SUPPOSE F R X, *Fig.* 35. to be a Ray given in Poſition, and the Rays F P X, F T X (whoſe Ratio of Incidence and Refraction are given) are to be inſerted at the Points F and X. Now feign alſo other Rays equally refrangible, as the Rays F P, F T to be incident in the Line F R, and deſcribe (by help of *Prop.* III.) their refracted ones R O, R K; and ſeek (by *Prop.* VIII.) the Centers Y and Z of their Radiations, and join Y X and Z X, meeting the refracting Surface in P and T. I ſay it is done; *viz.* F P X, F T X are the Rays, which were to be deſigned. For ſince by Hypotheſis the Difference of Refrangibility, and therefore the Diſtance of the Points T, R and P, is infinitely little, it is manifeſt, that the homogeneal Rays R O, P X are the neareſt to one another, and thence do diverge from the ſame Point of Radiation Y, ſo the Ray P X is rightly determined to paſs through the Center of

 Radiation;

Radiation ; and the like Reaſon holds in Relation to the Ray T X.

BUT ſince the Determination of the Angle P X T is to the End, that it may be known, how great, on Account of the unequal Refractions of diſſimular Rays, is the Confuſion of Objects ſeen by the Intervention of Refraction, and through how great a Space the Colours thence emerging are extended, as is manifeſt by conceiving F to be a lucid Point, which appears to the Eye placed in X to be dilated and diffuſed through the whole angular Space. P X T, that is comprehended by the Rays P X and T X, the moſt and the leaſt refrangible of all: I ſhall adjoin a few Things concerning its Magnitude. Feign the curve Line Y *f* Z to be deſcribed, in which lie the Centers of Radiations of all Sorts of Rays, incident in the Line F R, and ſo refracted in the Point R, that they may divaricate through the whole Angle K R O, and that Curve will not be unaptly likened to a lucid Object, whoſe viſible Angle, or apparent

rent Magnitude, to the Eye placed in X, is Y X Z, and its Diſtance from the ſame Eye, eſtimated from its middle, *f* X, and hence it follows.

1. That (ſince the apparent Magnitude of any viſible Thing is nearly reciprocally, as its Diſtance) the Point F remaining fixed, and the Point X taken any where in the Line R X, the Angle P X T or Y X Z will be nearly reciprocally as the Length *f* X; and hence the Interval R X being diminiſhed, the Angle P X T will be augmented, and its Quantity in any Diſtance of the Point X will be given, provided it was ever given in any one Diſtance.

2. Moreover the Angle O R K being known, any Angle P X T is known, by taking it in the Ratio to O R K, that R *f* has to X *f*, ſeeing Y R Z (to which O R K is equal) is the apparent Magnitude of the Object Y *f* Z in the Diſtance *f* R.

Since

SINCE therefore the Angle O R K for any Obliquity of the Rays incident in R F has been determined above in the *Schol.* to *Prop.* XI. and the Point f is not difficultly found, by making according to *Prop.* VIII. that it be R F. R f : : $\frac{AFq}{RF}$. $\frac{ADq}{RD}$, the Invention of the Angle P X T sufficiently appears.

BUT I observe by the by, that the aforesaid Curve Y f Z, in which are placed the Centers of Radiations of the Rays of all Sorts refracted in the Point R, is the vulgar or *Diocles's Cissoid* accommodated to a Circle whose Diameter is R E $= \frac{AR \times FRq}{AFq}$. That Circle R C E being described, let any right Line f B C be drawn perpendicular to R E, and terminated by the Circle in C and the Curve in f. And by reason of the analogous Sides of the similar Triangles R A D, R B f it will be A Dq, A R x D R : : B f q. B R x f R, and by applying the last Ratio to B R, it will become A D q. A R x

$DR :: \frac{Bfq}{BR}. fR$, and again by drawing the Consequents of the Ratios into Rf, and by applying to AR, there will arise $ADq. DR \times Rf :: \frac{Bfq}{BR}. \frac{Rfq}{AR}$. But it is $\frac{AFq}{FR}. \frac{ADq}{DR} :: RF. Rf$ as before, and the Consequents being drawn into DR and the Antecedents into FR, there arises $AFq. ADq :: FRq. DR \times Rf$, and alternately $AFq. FRq :: ADq. DR \times Rf$. Wherefore by connecting the Ratios agreeing to the same third, there will be had $\frac{Bfq}{BR}. \frac{Rfq}{AR} :: AFq. FRq$, and by drawing the Antecedents of the Ratios into BR, and the Consequents into AR, there will come out $Bfq. Rfq :: AFq \times BR. FRq \times AR$; and moreover by applying the latter Ratio to AFq, it will become $Bfq. Rfq :: BR. \frac{FRq \times AR}{AFq}$. But when I had made $RE. AR :: FRq. AFq$, it will be $\frac{FRq \times AR}{AFq} = RE$, and therefore $Bfq. Rfq :: BR. RE$, and by Division $Bfq. Rfq - Bfq\ (BRq) :: BR. BE$;

B E ; and from the Nature of the Circle B C is a mean Proportional between B R and B E, and therefore it is B R. B E : : B R q. B C q, and consequently B *f* q. B R q : : B R q. B C q, or B *f*. B R : : B R. B C, which manifests the Curve to be the *Cissoid*, as I proposed to shew.

THE Refractions at a Surface terminating two giving Mediums having been treated of, I now come to discover, what follows from the increasing the Rarity or Density of either Medium, or to compare amongst one another the Effects of different Mediums.

LEMMA III.

If from two Points D, G *(Fig. 36.) situated in any Line* A D, *there be drawn to two other Points* L, N *situated in its Perpendicular, the four right Lines* D N, D L, G N, G L ; *the Ratio of the Lines drawn to the remoter Point* N *acceeds more to an Equality, than the Ratio of those drawn to the* Fig. 36.

nearer

D R : : $\frac{B f q}{B R}$. f R, and again by drawing the Consequents of the Ratios into R f, and by applying to A R, there will arise A D q. D R × R f : : $\frac{B f q}{B R}$. $\frac{R f q}{A R}$. But it is $\frac{A F q}{F R}$. $\frac{A D q}{D R}$: : R F. R f as before, and the Consequents being drawn into D R and the Antecedents into F R, there arises A F q. A D q : : F R q. D R × R f, and alternately A F q. F R q : : A D q. D R × R f. Wherefore by connecting the Ratios agreeing to the same third, there will be had $\frac{B f q}{B R}$. $\frac{R f q}{A R}$: : A F q. F R q, and by drawing the Antecedents of the Ratios into B R, and the Consequents into A R, there will come out B f q. R f q : : A F q × B R. F R q × A R; and moreover by applying the latter Ratio to A F q, it will become B f q. R f q : : B R. $\frac{F R q \times A R}{A F q}$. But when I had made R E. A R : : F R q. A F q, it will be $\frac{F R q \times A R}{A F q} = $ R E, and therefore B f q. R f q : : B R. R E, and by Division B f q. R f q — B f q (B R q) : : B R. B E;

B E ; and from the Nature of the Circle B C is a mean Proportional between B R and B E, and therefore it is B R. B E : : B R q. B C q, and consequently B *f* q. B R q : : B R q. B C q, or B *f*. B R : : B R. B C, which manifests the Curve to be the *Cissoid*, as I proposed to shew.

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nearer Point L, *or it is* G N. D N > G L. D L.

FOR let it be G N. D N : : G L. R, and it will be G N q. D N q : : G L q. R q : : G N q — G L q. D N q — R q. Wherefore since it is D N > G N, or D N q > G N q, it will be D N q — R q > G N q — G L q. But it is G
† N° 46. N q — G L q = D N q — D L q † and therefore D N q — R q > D N q — D L q. That is, D L q > R q, or D L > R. And consequently, since it was supposed G N. D N : : G L. R, it will be G N. D N > G L. D L. Q. E D.

PROP. XIII.

There being supposed a common Sine of Incidence of Rays of a different Sort, the more different is the Density of the Mediums, the greater will be the Inequality of the Ratio of the Sines of Refraction.

Fig. 37. IN *Fig.* 37.) let F *c* be one of the least refrangible Rays incident in any manner on the Surface A *c*, and let its refract-

Pag.

Fig: 34.

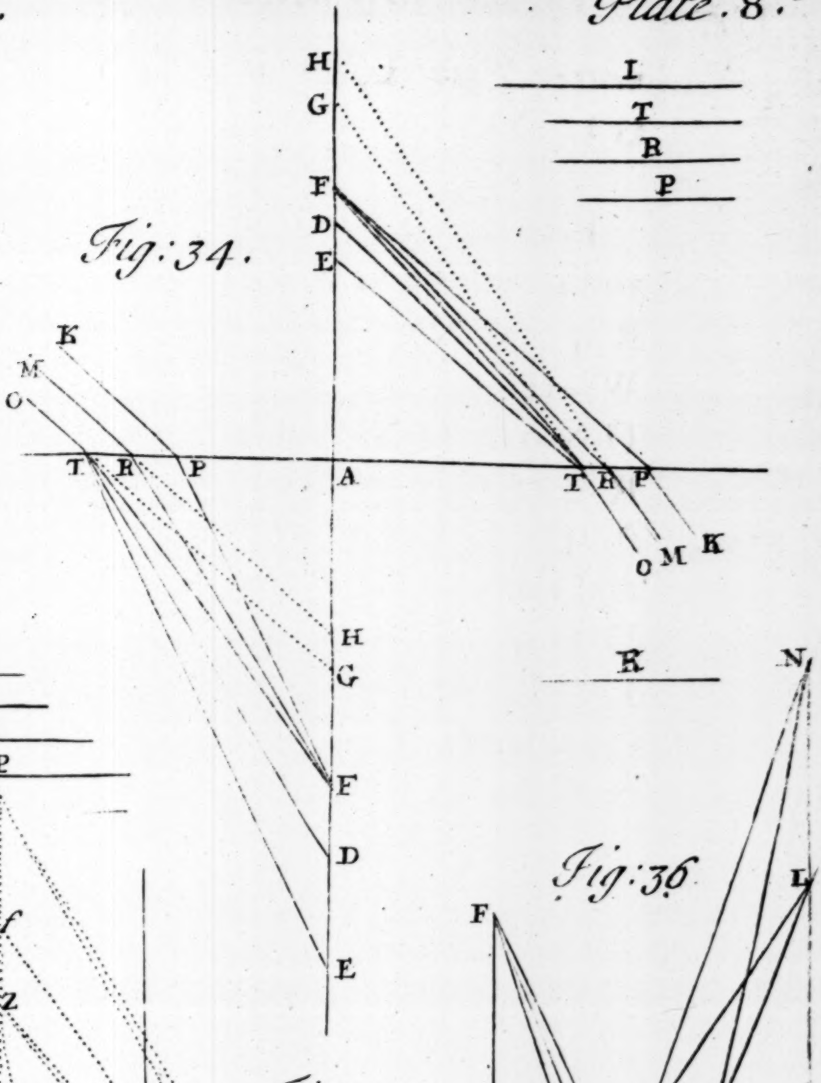
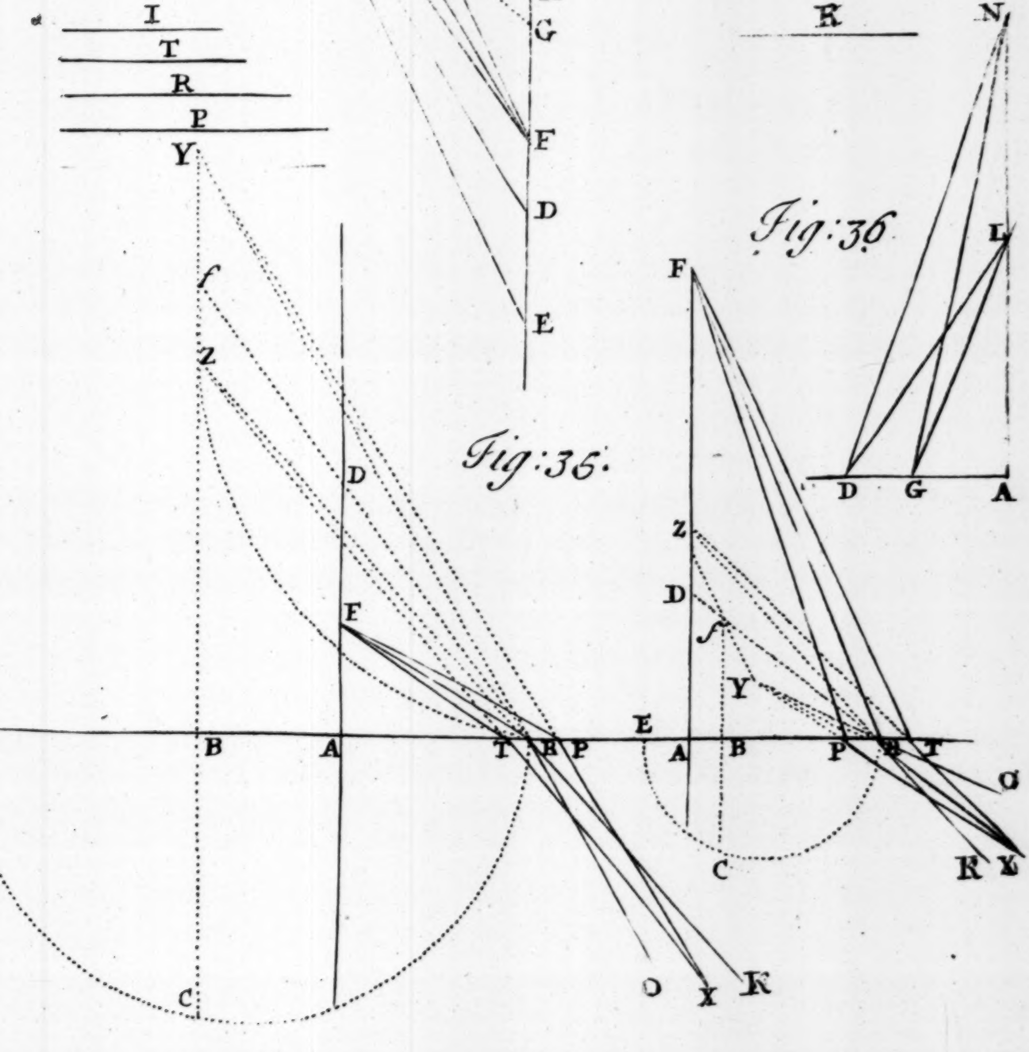


Fig: 36.

Fig: 35.



refracted Ray be $c\,l$, which continued backwards may cut the Perpendicular F A in f. Then let A e be taken, that F e be to F c in any given Ratio, ſuch as we have before deſcribed. [a] *viz.* with this Condition, that F e being reckoned one of the moſt refrangible Rays, its refracted one may diverge from the ſome Point f: This being done, if for the latter Medium another however denſe or rare be ſubſtituted, two the like Rays incident in the ſame right Lines F e, F c ought to be always ſo refracted, that they may diverge in like Sort from one and the ſame Point of that Perpendicular, [b] as from g towards l and n, it being ſuppoſed, that this latter Medium is of a Denſity more different from the former Medium, than the other latter Medium, that made the Rays diverge from f. It is therefore to be ſhewn, that the Inequality of the Ratio of the Sines of Refraction is greater in the latter than in the former Caſe. *Viz.* of the Ray F $c\,l$ the Sine of Incidence is to the Sine of Refraction as $f\,c$ to F c, that is, as 1 to $\frac{Fc}{fc}$,

[a] N° 44. 45. and 49.

[b] N° 45 & 49.

$\frac{Fc}{fc}$, and so of the Ray F en they are as 1 to $\frac{Fe}{fe}$, wherefore the Sines of the Refractions of the same Rays are amongst themselves as $\frac{Fc}{fc}$ to $\frac{Fe}{fe}$. And by a like Disquisition it will appear, that of the refracted Rays diverging from g, the like Sines of their Refractions are as $\frac{Fc}{c}$ to $\frac{Fe}{ge}$. It remains therefore to be proved, that between $\frac{Fc}{gc}$ and $\frac{Fe}{fe}$ there is a greater Disproportion, than between $\frac{Fc}{fc}$ and $\frac{Fe}{fe}$; that is (since $\frac{Fe}{fe}$ is $>\frac{Fc}{gc}$ [d]) it remains to shew, that it is $\frac{Fe}{ge}\cdot\frac{Fc}{\;} >$ $\frac{Fe}{fe}\cdot\frac{Fc}{fc}$. *Viz.* it is $ge.\,fe < gc.\,fc$ by *Lem.* III. and by taking the Reciprocals of the Ratios it will be $\frac{1}{ge}\cdot\frac{1}{fe} >$ $\frac{1}{gc}\cdot\frac{1}{fc}$, and by drawing the former Ratio into F e, and the latter into F c, there will arise $\frac{Fe}{ge}\cdot\frac{Fe}{fe} > \frac{Fc}{gc}\cdot\frac{Fc}{fc}$. and by Alternation $\frac{Fe}{ge}\cdot\frac{Fc}{gc} > \frac{Fe}{fe}\cdot\frac{Fc}{fc}$. Q. E D.

[d] *Lem.* III.

SCHOL.

SCHOL. The Demonstration is after the same Manner in the greater Letters (by which I have designed the Refractions, when the latter Medium is rarer than the former One) provided instead of the Sign $>$ there be every where understood the Sign $<$, and $>$ instead of $<$. You will also observe, that in this I have made the Density of the latter Medium only to be varied, but it comes to the same Thing, if you conceive the former Mediums to be successively varied, the latter not being changed, or which is all one, if the Refractions are contrarywise made out of the latter into the former Medium. For indeed to Rays falling on a Surface on either Side, there are the like Ratios of the Sines; but as to the exact finding the Ratio of these Sines, for any proposed Mediums, I have treated of this before, and I had not touched on the present *Proposition*, if the XV. *Proposition* to be delivered by and by had not required it.

LEMMA. IV.

Fig. 38. *With the Center* A, *any Distance* A D (in *Fig.* 38.) *let be described a Circle* D *g* G: *next with any Center* C, *distance* A C *let be described another Circle cutting the right Line* A D *in* B *and the former described Circle in* G, *then let the Arch* B G *be bisected in* F *and* F K *let fall perpendicular upon* B D. *These Things being thus done, I say, that* F K *so let fall perpendicular will bisect the said* B D.

For A F, A G, B F, F G and F D being joined, in the Triangles A F G, and A F D the Angles at A are equal on account of the equal Arches B F, F G, by which they are subtended. The Sides also about those Angles A D and A G are equal, *viz.* the Radius's of the same Circle, and they have the other Side A F common, wherefore also the third Sides F G and F D are equal. But B F is equal to F G, by reason of the Equality of the Arches, which they subtend. And consequently

BF

B F = F D, and the Triangle F K B = Triangle F K D, and thence B K = K D.

COROL. 1. Hence the right Line K F, which bisects the base Line B D, and insists on it perpendicularly, will also bisect the Arches B G of all the Circles passing through the two given Points A and B, and cutting somewhere in G the given Circle D G described with the Center A and Interval A G. And it will even bisect the Arches B G *g* in the other Point of Intersection *f*.

COROL. 2. The same Thing will happen, when A and B coincide, that is, when the Circles A F G touch the right Line A D in the Point A. Also B may be taken on the other Side of A. It may also be observed by the by, that the Angles B F K, B G D, which the Circle A B F makes with the right Line F K and Arch G D, are equal.

LEM-

LEMMA. V.

Fig. 39. *Four Lines* A *b*, AB, A *c and* AG (*Fig.* 39.) *being in any Circle from the same Point of the Circumference so inscribed, that it is* A *b*. AB :: A *c*. AG, *of all which let* A *b be the least, I say, that the Angle* BAG *is greater than the Angle* *b* A *c*.

For let another Circle AB be described, cutting the former in the Points A and B, whose Diameter let be to the Diameter of the Circle ABG, as AB to A *b*, both their Centers lying on the same Side of AB. Then with the Center A, distance AG describe a third Circle GH, meeting the second in *g*, and that Point *g* by Construction will lie somewhere between G and H; and consequently if A *g* be drawn, the Angle BAG will be greater than the Angle BA *g*. But the Angle BA *g* is = Angle BA *c*; by Reason that AB and A *g* are alike inscribed in the Circle AB *g*, as A *b* and A *c* are in A *b c*, *viz.* having the same Ratios, both

both amongst one another, (A *b*. A *c* :: A B. A G or A *g*) and to the Diameters of the Circles, in which they are inscribed. Since therefore it is B A G > B A *g* = *b* A *c*, it will be B A G > *b* A *c*. Q. E. D.

COROL. 1. Hence in any the same Sort of Rays, the greater the Refraction is, the greater will be the refracted Angle. In *Fig.* 27. where it is F R. R D :: F *r*. *r d*, it will be the Angle F *r d* > Angle F R D. *Fig.* 27.

COROL. 2. Hence also if it is A G. A B > A *c*. A *b*, it will be much more the Angle B A G > *b* A *c*. That is in general, the greater the Subtenses are, and at the same Time, the greater the Inequality is of their Ratio, the greater will be the Difference of the Angles, which they subtend ; and the same is to be understood of Sines and their Angles, as the Halves of Subtenses and their Angles.

LEMMA. VI.

Moreover if the Arch c d be taken equal to b c, and A D *be so inscribed in the Circle* A B D, *as it shall be to* A *d, as* A G *to* A *c; the rest remaining, I say, that the Arch* D G *will be greater than the Arch* G B.

For with the Centre A, Radius A D describe a Circle D K, E cutting the Circle A B *g* in K and the right Line A B in E, and let be drawn A K. Now since A K, A *g* and A B are inscribed alike in the Circle A B *g* K, as A *d*, A *c* and A *b* are in the Circle A *b c*; the Arch *g* K will be = Arch B *g*, wherefore *g* L being let fall perpendicular to B E, and produced till it cuts the Arch B D in F, that *g* L by *Lem.* IV. will bisect both the right Line B E and the Arch D B. But because *g* F by Construction lies out of the Circle *g* G, the Point F will fall between G and D. Wherefore D G > D F, or > F B and much more > G B Q. E. D.

COROL.

COROL. 1. Hence if the Arch *b d* does not consist only of two, but of any Number of equal Parts, the corresponding Parts of the Arch *b* D from the Termination *b* to the Termination D, will gradually exceed one another in Length. So that if the Arch *b c* has any commensurable Ratio to the Arch *g d*, it will be Arch G D. Arch B G > Arch *c d*. Arch *b c* : for indeed the Number of equal Parts measuring the Arches *bc* and *c d*, correspond to the like Number of unequal Parts, constituting the Arches B G and G D, whereof those in G D are all greater than the greatest Part of B G. Moreover, if *b c* has any incommensurable Ratio to *c d*, it will be in like Manner G D. B G > *c d*. *b c*. For the Similitudes of Ratios, which agree indefinitely to commensurable Quantities, on that Account do agree also to Incommensurables alike affected, as may be shewn from *Euclid*'s *Definition* of the like Ratios. But this is more easily understood, by imagining that the Quantities, which they call incommen-

ſurable, may be numbered by Parts indefinitely ſmall; and ſo in ſome Sort be reduced to the Nature of Commenſurables, eſpecially as to the Habitudes of Ratios. Conceive therefore the Arch *b c* to be divided into equal and indefinitely many Parts, and of theſe ſo many to be taken, as they ſhall differ leſs than by one Part (that is indefinitely little) from the Arch *c d*, and therefore they ſhall be thought, according to the uſual Manner, equal to it. Conceive alſo B D to be divided into equal Parts, correſponding (as I before defined) to the Parts of *b d*, and on Account of as many unequal Parts greater indeed in G D and leſs in B G, as there are equal ones in *c d* and *b c*, it will be G D. B G. > *c d. b c.*

COROL. 2. HENCE beſides by compounding it follows, that B D. B G > *b d. b c*, and alſo G D. B D > *c d. b d.*

Fig. 40. COROL. 3. It follows moreover, that the Subtenſes A *b*, A *c*, A *d*, A *e* being any Way drawn in *Fig.* 40. and other four

four A B, A G, A D, A E, whereof each obſerves to each of the other the ſame Ratio (*viz.* A B. A *b* :: A G. A *c* :: A D. A *d* :: A E. A *e*); If A E is the greateſt of them all, and A *b* the leaſt, it will be Arch E D. Arch G B > Arch *e d*. Arch *c b*. For by *Corol.* 1. of this *Prop.* it is E D. D G > *e d*. *d c*, and D G. G B > *d c*. *c b*, and much more E D. G B > *e d*. *c b*. Not otherwiſe it appears, that it is Arch E G. Arch D B > Arch *e c*. Arch *d b*, *viz.* by *Corol.* 2. of this, it is E G. D G > *e c*. *d c*, and D G. D B > *dc*. *d b*; and much more E G. D B > *e c*. *d b*. *Laſtly*, what has been ſaid of Subtenſes and their Arches, may be alſo underſtood of Sines and their Arches.

PROP. XIV.

Heterogeneal Rays being incident, out of a denſer Medium into a rarer, in the ſame given Line, on a Surface given in Poſition; the rarer the Medium is, into which they are refracted, the greater will be the Difference of Refraction.

Fig. 41. LET (*Fig.* 41.) F L be the Line, in which two Rays are incident on the Surface A L, whereof let the most refrangible Ray be refracted to P, and the least refrangible to T. I say, that if the rarer Medium was still more rare, that it should refract the most refrangible Ray to *p* and the least refrangible to *t*, then the Angle *p* L *t* would be greater than the Angle P L T. For let fall F A perpendicular to the refracting Surface, which may cut the refracted Rays continued backwards in G, C, D and E. Then in the refracting Surface let be sought such a Point as N, that it may be F N. D N :: F L. E L, and D N produced will be the refracted Ray of the least refrangible Ray, incident from F to
[a] No. 47. N [a]. Now when the Position of F L and F N is supposed to be such, that the refracted Rays D L and D N of the most refrangible Ray incident in F L, and of the least refrangible Ray incident in F N, do diverge from the Point D, which is situated in the Perpendicular F A, for that Cause, although

though the Rarity of the Medium, in which the Refraction is performed, were different than is ſuppoſed, yet of the like Rays incident in the ſame Lines F N and F L their refracted ones would always diverge from ſome Point, which is placed in the ſame F A, as is ſhewn in what went before [b]. So when the Rarity of the ſaid Medium is ſuppoſed to be ſuch, that the moſt refrangible Ray incident in F L is refracted from any Point G; then the leaſt refrangible Ray incident in F N will be refracted from the ſame Point G. But when the moſt refrangible Ray was ſuppoſed to be refracted from the Point G, then alſo the leaſt refrangible Ray incident in the ſame Line F L was ſuppoſed to be refracted from the Point C. Wherefore it is G N. F N :: C L. F L [c], and beſides ſince I ſuppoſed it before to be F N. D N :: F L. E L, it will be by Equality G N. D N :: C L. E L. But by *Lemma* III. it is G N. D N > G L. D L, and conſequently C L. E L > G L. D L. Wherefore if a Line B L be ſo drawn, that it may be C L. E L :: B L. D L, it

[b] N° 49.

[c] N° 25 and 47.

it will be D L > G L, on Account of the greater Ratio, which it has to D L; and moreover C L will be greater than B L, becauſe it is E L > D L, and therefore the Point B will fall between G and C, and the Angle G L C will be > Angle B L C; but ſince it is C L. E L :: B L. D L, or alternately B L. C L :: D L. E L, the Angle B L C will be greater than the Angle D L E (*Lem.* I.) and much more the Angle G L C > Angle D L E. Q. E. D.

PROP. XV.

Heterogeneal Rays being incident out of a denſer Medium into a rarer in the ſame given Line upon a Surface given in Poſition; the denſer the Medium is, out of which the Rays are incident, the greater will be the Difference of the Refraction.

FOR (on account of the greater Refractions) the greater will be the Sines of the Refractions, in reſpect to a given Circle, to which they are referred, and at the ſame Time the greater will be

be the Inequality of the Ratio of those Sines, by *Prop.* XIII. and consequently the greater will be the Difference of the Angles, which they subtend, by *Corol.* 2. to *Lem.* V. that is, the greater will be the Difference of Refraction, Q. E. D.

PROP. XVI.

Heterogeneal Rays being incident out of a rarer Medium into a denser in the same given Line upon a Surface given in Position; the rarer the Medium is, out of which the Rays are incident, the greater will be the Difference of the Refraction.

LET (in *Fig.* 42.) A D be a Surface, on which two Rays are incident in the same given Line I X, whereof one the most refrangible let be refracted to P, and the other the least refrangible to T. I say, that if the Medium, out of which the Rays are incident, were still rarer, that it might refract the said Rays still more, as the most refrangible one towards *p*, and the least refrangible towards *Fig.* 42.

wards *t*, then the Angle *p* X *t* would become greater than P X T. Which I thus gradually demonſtrate.

CAS. 1. LET us in the firſt Place ſuppoſe, that the right Line I X, in which the Rays are incident, is the moſt oblique to the refracting Surface, and let any right Line P D be drawn, perpendicular to the ſame Surface, and cutting the refracted Rays in the Points T, P, *t*, *p*, and let I X be produced, till it cuts P D in *f*. Then in the Line A D let be ſought a certain Point B on this Condition, that B *f*, B P being drawn, it may be X *f*. X T : : B *f*. B P. It appears therefore, that if the leaſt refrangible Ray is incident in B, tending towards *f*, it ought to be refracted towards P, for ſince by Hypotheſis it is B P. B *f* : : X T. X *f*, that is, the Sines of its Incidence and Refraction, as the Sines of Incidence and Refraction of another the leaſt refrangible Ray I X *f*: Wherefore if we ſuppoſe thoſe Rays to recede backwards, *viz.* one of the leaſt Refrangibles from T to X, and the other from P to B, and

and the moſt refrangible Ray from P to X, the refracted ones of them all tend from the Point *f*; for it is a known *Theorem*, that of a Ray being incident backwards in its refracted Ray, the Incident becomes the refracted. Now when the difform Rays P B, P X, flowing from the ſame Point P, are refracted from the ſame Point *f*, which is ſituated in the Perpendicular P D, the Proportion between P X and P B being once known, if from any other Point of the ſame Perpendicular there be drawn to the refracting Surface two Lines having the ſame Ratio, that is, that one of them deſigning the moſt refrangible Ray be to the other, that deſigns the leaſt refrangible Ray, as P X to B P: Then their refracted Rays (from what has been ſhewn before [a]) will diverge alſo from ſome Point, that is ſituated in the ſame Perpendicular P D; however rare the Medium is ſuppoſed, on the Side of the Ray I X, provided the other Medium on the Side of the Ray P X retains the ſame Denſity. As if the moſt refrangible Ray is incident in *p* X and is refracted from *f*, *viz.* the

[a] N°. 45.

the Medium towards I X being now supposed rarer than before, then the right Line *p b* being so drawn, that it be P X. B P : : *p* X. *p b*, the least refrangible Ray *p b* would be refracted also from the same *f*, whence it follows, that *p b* is to *f b*, as the Sine of Incidence of the least refrangible Rays to the Sine of Refraction [a] But in the Ratio of those Sines is also *t* X to *f* X, because the inflected Line I X *t* denotes a Ray equally refrangible, whose Part I X produced passes through the same Point *f*; wherefore it is *p b*. *f b* : : *t* X. *f* X. But since the Ray I X is supposed to be the most oblique to the refracting Surface, or inclined in an infinitely small Angle, so that the right Line D *f* ought to be esteemed as infinitely small or nothing, it follows, that D X = X *f*, D B = B *f*, and D *b* = *b f*: Which Values substituting for X *f*, B *f* and *b f* in the above recited Proportions B P. B *f* : : T X. X *f*, and *p b*. *f b* : : *t* X. *f* X, there will come out B P. B D : : X T. X D, and *p b*. D *b* : : *t* X. D X. From which it appears, that the right Lines B P and X T;

[a] N° 47.

X T ; *b p* and X *t* are parallel, and the Angles B P X and P X T ; *b p* X and *p* X *t* are equal, but by Hypothesis it is P X. B P : : *p* X. *p b*, and therefore the Angle *b p* X > Angle B P X by *Corol.* 1. *Lem.* V. that is Angle *p* X *t* > Angle P X T. Q. E. D.

CAS. 2. BUT the incident Rays making with the refracting Surface an Angle of a definite Magnitude, the *Proposition* thus appears. Let H X be (in *Fig.* 43.) the right Line, in which the Rays are incident ; and when they come out of a less rare Medium, let X M be the least refracted, and X N the most refracted Ray. But when they come out of a more rare Medium let X *m* be the least refracted, and X *n* the most refracted Ray. Let also be used the most oblique incident Rays I X with their refracted ones X T, X P, X *t* and X *p*, such as I have now described. *viz.* so, that when the Rarity of the anterior Medium is so great, that it makes the Rays H X to be bent towards M and N, then let it also bend the like Rays I X towards T and P, but *Fig.* 43.

when its Rarity is ſo much greater, as that it compels thoſe towards *m* and *n*, then at the ſame time let it compel theſe towards *t* and *p*. Let moreover A P D be a Circle, deſcribed with the Center X and any Interval A X, that may cut theſe refracted Rays in T, P, M, N, *t*, *p*, *m*, *n*, from which to the Perpendicular B X let fall the Sines of the Refractions T B, P C, M F, N G, *t b*, *p c*, *m f*, *n g*, and from the Law of Refractions it will appear, that it is T B. P C : : M F. N G, and *t b*. *p c* : : *m f*. *n g*; and farther by Hypotheſis and Conſtruction it will appear, that T B is the greateſt and *n g* the leaſt of thoſe Sines. And conſequently by *Corol.* 3. *Lem.* VI. it is Angle T X P. Angle M X N > Angle *t* X *p*. Angle *m* X *n*, and by Permutation it is Angle T X P. Angle *t* X *p* > Angle M X N. Angle *m* X *n*. But from what was ſhewn in the firſt Caſe, the Angle T X P is < Angle *t* X *p*, wherefore and much more it will be the Angle M X N < Angle *m* X *n*. Q. E. D.


g
f
F
E
G
A


F
E
D
C
B
G
A

Pag:142.

Plate 9th

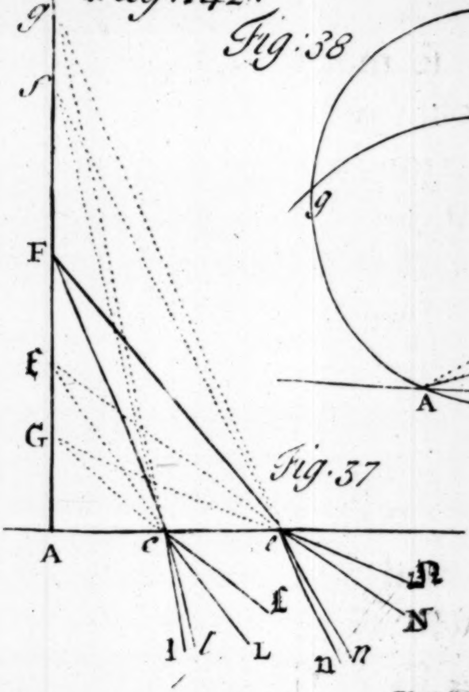


Fig:37

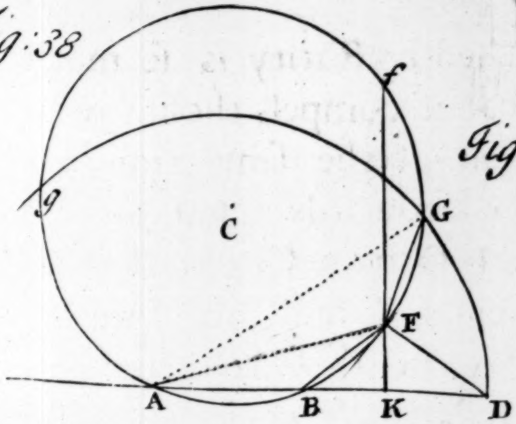


Fig:38

Fig:40

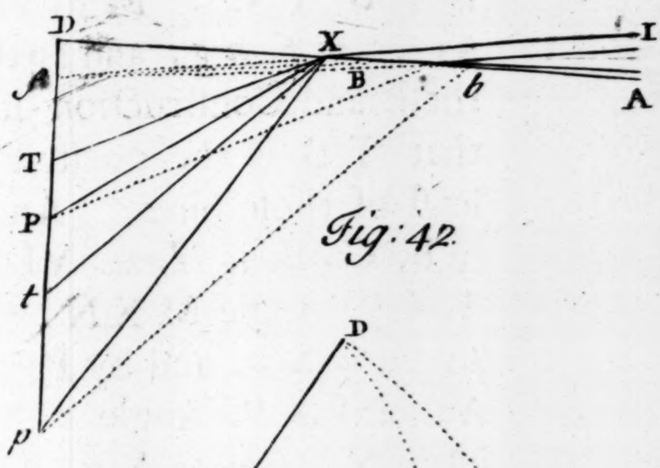


Fig:42

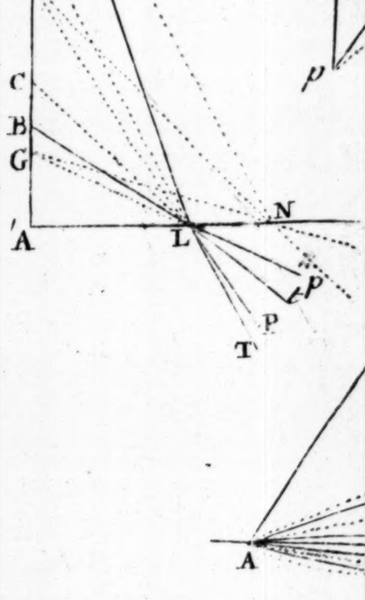


Fig.41.

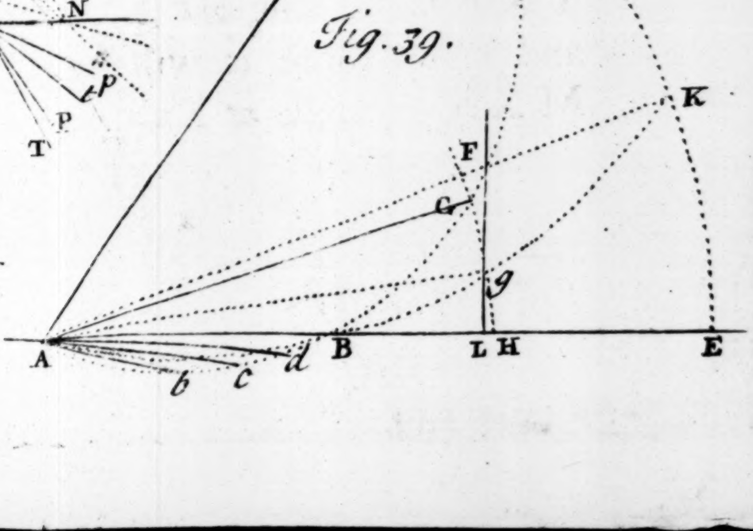


Fig.39.

PROP. XVII.

Heterogeneal Rays being incident out of a rarer Medium into a denser, in the same Line, upon a Surface given in Position; the denser the Medium is, on which the Rays are incident, the greater will be the Difference of the Refractions unto a certain Limit, and afterwards it will be less perpetually.

FOR if the latter Medium exceeds very little the anterior in Density, so that it makes indefinitely small Refractions, the Difference of Refractions will be also indefinitely small, and therefore less than it would be, if the latter Medium were supposed more dense, that the Refractions might become greater. Wherefore the Density of the latter Medium being increased, the said Difference of Refractions will be increased; but if its Density be increased *in infinitum*, the Refractions also will be increased as much as possible. That is, until all the refracted Rays emerge perpendicularly, the Angles of Refractions and

and their Differences then altogether vanishing. Wherefore the Difference of Refractions is again diminished, until it has vanished into nothing.

SCHOL. ALTHOUGH the Determination of its Limit, where the Difference of Refraction becomes the greatest, may afford more Pains and Labour, than Utility; however, since it may perhaps be thought of some Moment to know the Density of a Medium, that may make by the Rays refracted into it the most conspicuous Colours, I shall not think much to design this. And that in the first Place, when the Incidence is most oblique.

Fig. 44. CAS. I. LET (in *Fig.* 44.) I X be the common Way of Rays falling the most obliquely upon the Surface A X separating any Mediums, and let their refracted Rays be, as before, X*p* and X*t*; and let a right Line *p t* be drawn parallel to the aforesaid Surface, that may meet those Rays in *p* and *t*, from which to A X the Perpendiculars *p* C, *t* E being let fall, let C E be bisected

sected in D, and with the Center D, distance D X let a Circle be described cutting C*p* in P, and E *t* in T, and let XP and X T be joined. I say, that when the Density of the latter Medium is such, that of the Rays incident in the Line I X it refracts the most refrangible ones to P, and the least refrangible to T, then the Angle P X T will become the greatest. For however dense the latter Medium is supposed, the refracted Rays will so cut the Lines C P and C T in the Points *p* and *t*, that the right Line *pt* may be parallel to A X. Wherefore if the Line D *r* be drawn, that may bisect all the Lines *pt*, the Center of any Circle passing through *p* and *t*, will always lie in the same D*r*. But the Angle *p*X*t* is the Angle in the Segment of a Circle passing through the Points *p*, *t* and X; which Angle therefore will be the greatest, when the like Circle is the least; because the Ratio of the Subtense *pt* to the Dimensions of the Circle becomes then the greatest. But that Circle becomes the least of all, when its Center falls in D. For it then has for its Semidia-

meter X D the least of the refracted Lines, which can be drawn from X to R D. The Angle *p* X *t* therefore is then the greatest, when the Center of the Circle passing through the Points *p*, *t* and X falls on D. And consequently, when the Circle X P T and Angle P X T are of this Kind, the *Proposition* is manifest.

HENCE it appears by the by, that this Angle P X T then also becomes the greatest, when the Density of the latter Medium is such, that the Angle of Refraction of the mean refrangible Rays, being incident most obliquely in I X, is Half a right Angle; and continually becomes less, the more the Angle of Refraction deviates (by Excess or Defect) from Half a right Angle. As if the Refractions out of Air into Water, into Glass and into Crystal were compared, it will appear from Calculation, that, when the Angle of Incidence is Ninety Degrees nearly, then the Angle of Refraction into Water will be greater than Half a right Angle, into Glass it will be less.

Wherefore Water is leſs denſe, and Glaſs more denſe, than that they may make the Angle P X T the greateſt. And therefore ſince Cryſtal is ſtill denſer, it will make that P X T leſs, than Glaſs would make it. And ſo Glaſs, although it refracts leſs, yet in that Caſe it will more diſſipate from one another the heterogeneal Rays refracted into it, than Cryſtal; and by that Means it will project Colours more ſpread, on its oppoſite Surface. But theſe are moſt difficult to try, becauſe Glaſs and Cryſtal differ little in their Denſity, nor can they be had ſufficiently thick; and if they could, then on Account of too great Denſity, they would not be perſpicuous enough.

CAS. 2. But if the Line, in which the Rays are incident, is not the moſt oblique, the *Problem* becomes ſolid. But I intend to ſhew a Way, whereby, its Conditions being ſomewhat changed, it may be reduced to a Plane *Problem*. It muſt therefore be underſtood, that ſince between the extreme, or greateſt difform Rays, there

are innumerable intermediate ones, which by continually ſucceſſive, and infinitely ſmall Degrees, are ſome refracted more than others; the Difference of the extreme Rays will be made up of ſimilar Differences of the intermediate ones, infinite in Number and ſmalneſs. Now, the Proportions of thoſe infinitely ſmall Differences being known, we may from thence make a Judgment of all the Aggregates together, or of the finitely ſmall Differences, ſuch as intercede the Refractions of the extreme Rays; eſpecially ſince thoſe Differences are very ſmall. So it being known, that the infinite ſmall Differences are increaſed, diminiſhed, or at the ſame Time become greateſt, or leaſt; it will be concluded, that the Sum of all is in the ſame Manner increaſed, diminiſhed, is the greateſt or the leaſt. But if they are not all at the ſame Time the greateſt or the leaſt, yet the Sum may be eſteemed the greateſt or the leaſt, when it happens to an intermediate Part So the Breadth of all the Colours may be then thought the greateſt, when it happens

to

to the green. Now although the proposed *Problem*, when the Question is about finite small Parts, is solid; yet if it be made of infinitely small Differences, it may be reduced to a plane one. But in solving it I shall not take much Pains; I shall only briefly shew, by what Means a Calculation in this and others the like is to be performed, that an Equation may be come at, from which the greatest of the infinitely small Angles may be discovered. And farther from the same Fountain, I shall determine the Proportions of the Differences of Refraction in respect of different Mediums, which in the four preceding Propositions I did but describe in general.

IN the first Place therefore a Rule or Equation is to be investigated, whereby from any given refracted Ray whatever, another refracted Ray making with it an infinitely small Angle may be known. Rays being incident as before in the most oblique Line I X (*Fig.* 45.) out of a Medium of a given Density into a Medium of any Den- *Fig.* 45.

ſity whatever, let XR and X*r* be two refracted Rays, whereof let one XR be a little more refrangible than the other X*r*, yet by an infinitely ſmall Difference. And let be drawn any right Line R*r* meeting theſe in R and *r*, and parallel to the refracting Surface. To which Surface alſo let fall the perpendiculars RD, *rd*, which feign to have a given and finite Diſtance from X; but from one another an infinitely ſmall one. But conceive the Lineola R*r* with the Rays paſſing through R*r*, to lie more or leſs from XD (as in the preceding) according to the various aſſumed Denſity of the latter Medium. Now if the right Line DR cuts the Rays X*r* in M, and IX in K, ſince the infinitely ſmall Triangle RM*r* is ſimilar to the Triangle DMX, from which the Triangle KRX differs but by the infinitely ſmall Differences RXM, and DXK, which do not make a Diſſimilitude, the Triangles alſo RM*r* and RDX ought to be looked upon as ſimilar; and therefore the Perpendiculars KL and RN being let fall, it will be DX. LR ::

LR :: Rr. MN. And consequently since it is $LR = \frac{XRq - XKq}{XR}$; for it is XR. KR $(= \sqrt{XRq - XKq})$ :: KR. RL, it will be also $MN = \frac{XRq - XKq}{XR \times XK}$ into Rr. which is the Difference between XN, or XR, and XM. And then it will be $XM = XR - : \frac{XRq - XKq}{XR \times XK}$ into Rr. There is therefore found the Relation between XK, XM and XR, when the Angle IXA is infinitely small. Moreover however oblique it is supposed, those Lines XK, XM and XR will observe the same Relation, for they are reciprocally, as the Sines of Incidence; and Refraction; and therefore there is also found the Relation between them for any Obliquity of the incident Ray IX. And so XK and XR being known or any ways assumed at Pleasure, thence at the same Time XM is known. Which is the first Thing I proposed to determine.

WHEREFORE let IX be a Line making with the refracting Surface any

given Angle AXI, and the rest remaining it will be $MN = \frac{XRq - XKq}{XR \times XK}$ into Rr. Farther it is $RD (= \sqrt{XRq - XDq})$. $XD :: MN . RN$; and consequently it is $NR = \frac{XRq - XKq \text{ into } Rr \times XD}{XR \times XK \times \sqrt{XRq - XDq}}$. But if NR is divided by XR, there will come out the Sine of the Angle RXN with regard to a Circle, whose Semidiameter is Unity. Wherefore since that Angle and its Sine are the greatest, to determine the greatest Angle, the greatest Quantity NR must be sought, that is, the greatest $\frac{XRq - XKq \text{ into } Rr \times XD}{XRq \times XK \times \sqrt{XRq - XDq}}$ or (a Division being made by the given Quantity $\frac{Rr \times XD}{XK}$) there must be sought the greatest $\frac{XRq - XKq}{XRq \times \sqrt{XRq - XDq}}$ Which may be done by the well known Method *de Maximis & Minimis*, and there will come out $XRqq = 3 XKq \times XRq - 2 XKq \times XDq$; the Construction of which Equation is thus.

Fig. 46. From any Point (*Fig.* 46.) of the incident Ray IX let fall the Perpendicular IA, and in it take AF = AX, and XI being

being produced to B that I B may be $= \frac{1}{2}$ I X, upon B X describe a Semicircle B E X, in which inscribe X E = X F, then X B being produced to C, that B C may be = B E, upon C X describe a Semicircle C G X, which a Perpendicular erected to its Diameter at I may cut in G. Lastly with the Center X and Interval G X let be described the Arch G H cutting A I produced in H. Let H X be drawn and produced towards R, and R X will be the refracted Ray of I X, when the Density of the latter Medium is so great, that the Difference of Refraction R X M becomes the greatest of all. Which being found, the Density of the latter Medium causing such a Refraction will be easily given. Conceive therefore the Rays X R and X *r* to be mean refrangible ones, but in a different Degree, and the latter Medium so found will make not only between these, but also between the extreme, or the greatest difform Rays, nearly the greatest Difference, it possibly can.

BUT

BUT if there be required the Proportion of the like Sort of Differences to a various Rarity or Denſity of Mediums, this will be eaſily determined, from what has been now ſhewn, provided they are ſuppoſed infinitely ſmall. So the Rarity or Denſity of the latter Medium only being varied, that the Rays incident in I X are now refracted to M and R, then to *m* and *r*; and any Line DK being drawn perpendicular to D X, that may cut them in K, M, R, *m* and *r*, the infinitely ſmall Angle M X R will be to the like Angle *m* X *r*, as $\frac{XRq - : XKq}{XRq \times RD}$ to $\frac{Xrq - : XKq}{Xrq \times rD}$. But if the Rarity or Denſity of the former Medium is varied, the latter Medium not being changed: The Analyſt will eaſily

Fig. 45. diſcover, that (in *Fig.* 45.) it is M N $= \frac{XRq - : XKq}{XKq}$ into R *r*, and therefore

Fig. 47. that (in *Fig.* 47.) it is the Angle MXR. Angle *m*X*r* $:: \frac{XRq - : XKq.}{XR \times RD} . \frac{Xrq - : XKq,}{Xr \times rD}$ for it is not the ſame Thing, whether the Rarity or Denſity of the former Medium or of the latter Medium be varied, as ap-

appears from what has been before ſhewn.

THE preceding *Propoſitions* relate to the Diffuſion of Light flowing from a far. In the two following is treated of the Refraction of Light as proceeding from near at Hand.

PROP. XVIII.

Heterogeneal Rays being refracted from a given Point to a given Point, by a Surface given in Poſition; the more denſe the denſer Medium is, the greater will be their Inclination to one another on the Side of both the Mediums to a certain Limit, and afterwards it will be the leſs.

FOR when its Denſity is not greater than the Denſity of the other Medium, that the Refractions may be infinitely ſmall, then the Difference of Refraction will be alſo infinitely ſmall, and therefore will be increaſed, by increaſing the Denſity. But if its Denſity be increaſed *in infinitum*, then of all the Rays inci-

incident upon it, the refracted ones do emerge perpendicularly [a], and on the contrary the Perpendiculars alone can enter the rarer Medium out of the denser; whence all the Rays refracted from a Point to a Point will then proceed in the same Lines, or coincide, and so the Difference of Refraction will again vanish into nothing.

[a] N° 42 & 45.

PROP. XIX.

Heterogeneal Rays being refracted from a given Point to a given Point by a Surface given in Position; the more rare the rarer Medium is, the greater will be their Inclination to one another.

ON the Side of both Mediums, let A T be a Surface, so refracting the
Fig. 48. difform Rays F T X and F P X, that they flowing from the same Point F, they may again meet in the same Point X. I say if the former Medium were rarer, that the aforesaid Rays might be still more refracted, as F T X into F *t* X, and F P X into F *p* X, that the Angle *p* X *t*

X *t* would be greater than the Angle P X T, as alſo the Angle *p* F *t* greater than the Angle P F T.

To abbreviate the Demonſtration of the firſt Caſe, let us ſuppoſe the Rays to be the leaſt difform, that on account of the infinitely ſmall Difference of Refraction, they may make the Angles P X T and *p* X *t* infinitely ſmall [a]; then let be drawn T K the refracted Ray of a Ray conform to F P X, that the infinitely ſmall Angle K T X may be the Difference of Refraction of the Rays incident in the ſame Line F T; and after the ſame manner let be drawn *t k* the refracted Ray of a Ray conform to F *p* X, that the infinitely ſmall Angle *k t* X may be the Difference of Refraction of the Rays incident in the ſame Line F *t*. It appears therefore, that ſince the Ray F *t* is obliquer than F T, and is alſo incident on a denſer Medium, the Angle *k t* X will be greater than the Angle K T X. Farther let be produced K T and *k t*, till they cut in the Points D and *d* the Line F A, which is perpendicular to the Plane A T, and let

[a] Conſult *Caſ.* 2. of *Schol.* to *Prop.* XVII.

let them be produced beyond to f and g, so that it may be $\frac{FAq}{FT}$. $\frac{DAq}{DT}$: : T F. T f, and $\frac{FAq}{Ft}$. $\frac{dAq}{dt}$: : t F. $t\,g$, and the Points f and g so found will be the Foci of the Rays F T X, and T t X by *Prop.* VIII. *Cas.* 2. and X f. T f : : Angle K T X. Angle P X T; as also Xg. $t\,g$: : Angle K t X. Angle p X t. (*Cas.* 3. *Schol. Prop.* XII). These Proportionalities indeed are not altogether true, when the aforesaid Angles made by the Difference of Refraction are supposed to be of any definite Magnitude. But they approach the nearer to the Truth, the less these Angles are made, so that in infinitely small Angles they ought to be looked upon as accurately true. Now since by Hypothesis A t is $>$ A T, it will also be X $t >$ X T, as also $t\,g >$ T f, as appears from the Determination of the Points g and f given above. Wherefore it is $t\,g$. T f $> t$ X. T X, or by Permutation $t\,g$. t X $>$ T f. T X, and by compounding $t\,g$. Xg $>$ T f. X f, that is, by substituting Ratios equal to these, the Angle p X t. Angle k t X $\geq$ Angle P Xt. Angle

Angle K T X, and by Permutation *p* X *t*. PXT > *k t* X. K T X, as has been ſaid; and therefore much more is the Angle *p* X *t* > Angle P X T. Q. E. D.

BUT from hence may be made a Conjecture of the latter Caſe, that the Angle *p* F *t* is always > Angle P F T; for it would require a far more difficult Demonſtration, and yet I am weary of having beſtowed many Words upon them already; let theſe therefore ſuffice for the Refractions of a ſingle Surface.

Of the AFFECTIONS *of* RAYS *twice refracted.*

BUT if the Refraction be double, as it happens in Priſms, whoſe Phænomena I chiefly intended to explain, the Affections of Rays ſo refracted, are ſo manifeſt from the preceding, that it may ſeem a ſmall Matter to treat about them. Of parallel Surfaces indeed nothing farther occurs to be obſerved, than that the latter Surface as much refracts the Rays, as the former, but towards

wards a contrary Way. Of inclined ones what follows is to be obſerved.

PROP. XX.

Homogeneal Rays diverging to a Priſm, after both Refractions proceed to diverge.

It is manifeſt by *Prop.* VII.

And the ſame is to be underſtood of parallel or converging Rays, *viz.* that after both Refractions they will continue parallel or converging.

SCHOL. But if the Point, from which any infinitely near Rays after both Refractions do diverge, or the Place of the Image ſeen thro' the Priſm ſhould be deſired, its Invention is manifeſt from the *Scholium* of *Prop.* VIII. But that it may be readier done by Conjecture, this Mechanick *Theorem* may be uſed. That the Image will appear at about that ſame Diſtance behind the Priſm, as the Object has, whoſe Image

Pa

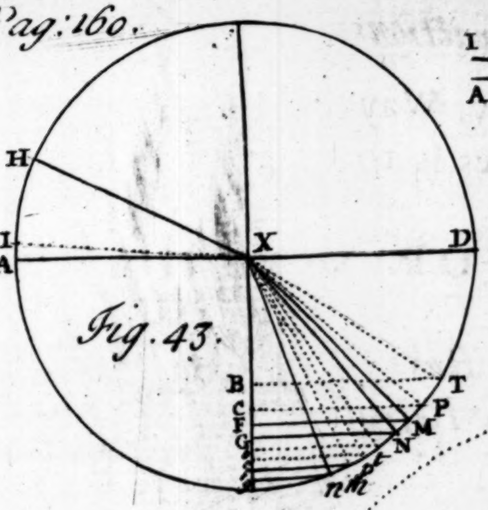


Fig. 43.

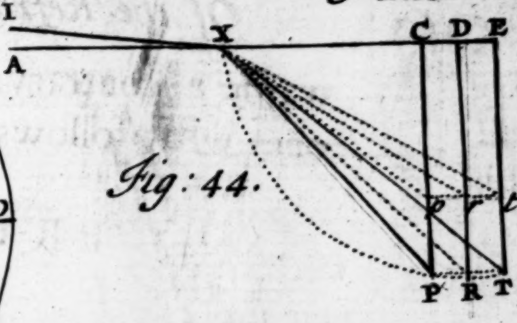


Fig: 44.

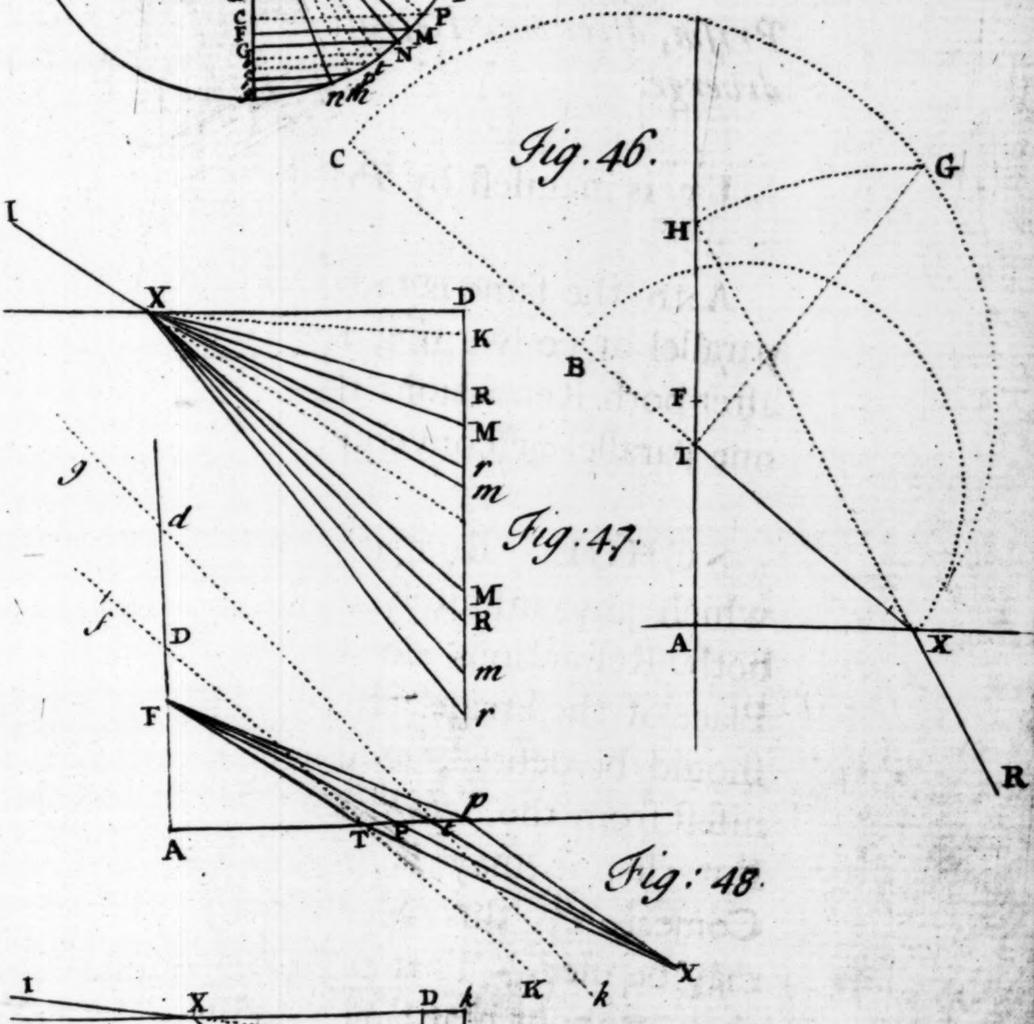


Fig. 46.

Fig: 47.

Fig: 48.

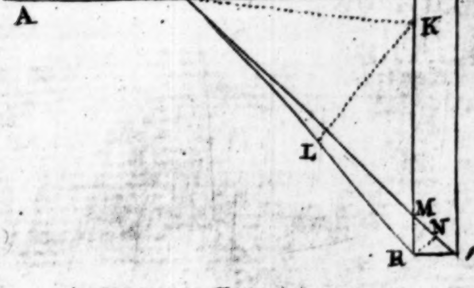


Fig: 45.

mage it is, provided the Refractions on each Side be not very unequal.

PROP. XXI.

Of Heterogeneal Rays diverging to a Prism, some after both Refractions will converge.

THIS appears from *Prop.* X and XII: *viz.* Of those, that lie in a Plane perpendicular to both the refracting Planes, the greater refrangible Rays from a somewhat more oblique Incidence will meet with the less refrangible ones, and the same Thing happens of almost innumerable others.

PROP. XXII.

Of Rays therefore so refracted from a Point to a Point, or from the Object to the Eye, some will pass gradually nearer to the Vertex of the Prism than others, according as they are more and more refrangible.

FROM *Prop.* X. whence the Orders of Colours are defined, of which hereafter.

PROP. XXIII.

The greater the vertical Angle of the Prism is, cæteris paribus, *the greater will become the Difference of Refraction, and thence the Appearance of the Colours the more distinct.*

AND this is manifest from *Prop.* II.

PROP. XXIV.

The denser the Matter of the Prism is, or the rarer the incompassing Medium is, cæteris paribus, *the greater will be the Difference of Refraction, and thence the Appearance of the Colours will be the more manifest.*

THE latter Case is evident from *Prop.* XIV and XVI. But the former Case, that it may not be called in Question from *Prop.* XVII. I thus shew.

Conceive a more refrangible Ray P D and a least refrangible one T D, so to fall on a Prism at any the same Point D, that their refracted Rays may proceed in the same Line D *d*, and lastly being refracted at *d* may diverge towards *p* and *t*. Which being supposed, it appears by *Prop.* XV. that the Angle *p d t* will be increased, from the Density of the Prism being increased; and the Conclusion is the same of the Angle PDT, provided consimilar Rays are conceived to recede in the same Lines. The Assertion therefore is manifest of Rays coinciding in the Prism, and thence also of parallel Rays. *Fig.* 49.

LEMMA VII.

Three homogeneal Rays b I, g I, d I, refracted out of a denser Medium into a rarer one by the Surface I K, *if the Differences of the Incidences b* I *g, g* I*d are equal, the Sum of the refracted Angles made by the extreme Rays will be more than double the refracted Angle made by the intermediate ones. That is, the refracted Rays being drawn* *Fig.* 50.

 back

back to B, G *and* D, *I ſay, that the Angle* B I *b* + D I *d is* > 2 Angle *g* I G.

FROM any Circle A D G being deſcribed touching the refracting Surface in I, whoſe Diameter let be A I, and which may cut the ſaid Rays in *b*, *g*, *d*, B, D, G. When indeed the Angles *b* I *g* and *g* I *d* are equal, the Arches alſo *b g* and *g d* will be equal. But A *g*, A *b* &c. being drawn: A *b*, A *g*, A *d* will be the Sines of the Incidences, and conſequently amongſt one another, as are A B, A G, A D the Sines of the Refractions. Wherefore (by *Lem.* VI.) the Arch D G is greater than the Arch G B, and thence $2\,gG > 2\,gG - GD + GB = gD + gB = gD - gd + gb + gB = Dd + Bb$. That is $2\,gG < Dd + Bb$, or Angle B I *b* + Angle *d* I D > 2 Angle *g* I G. Q.E.D.

PROP. XXV.

Homogeneal Rays being refracted by a Priſm, the Angle, which the incident and emerging Rays comprehend, then

then becomes the greatest, when the Refraction is equal on both Sides.

Let A B C be a Prism, G R S N a Ray refracted equally at R and S. Let I P Q L be another Ray refracted unequally, more at P, less at Q, and let these Rays be produced, until they meet one another, I P and Q L in T, but G R and N S in V. I say, that the Angle R V S is greater than the Angle P T Q. Which, that it may appear, conceive Rays proceeding each way in the Lines P Q and R S to go out of the Prism on each Side and so to be refracted out of a denser Medium into a rarer one. For in the Triangles C P Q, C R S, since the Angle C is common, the Sums of the other Angles will be equal, and therefore, since C R S is Isosceles, the double of the Angle C R S will be equal to the Angles C P Q + C Q P. Wherefore the Incidence of the Ray Q P at P, is as much greater than the Incidence of the Ray R S at S, as the same Incidence is greater than the Incidence of P Q at Q. The Differences therefore of three Incidences are equal, and consequently

 by

by the laſt *Lemma*, the Sum of the refracted Angles made by the greateſt and leaſt Incidence, will be greater than double the refracted Angle made by the middle Incidence. That is, Angle Q P T + Angle P Q T > 2 Angle R S V or > Angle R S V + V R S. Therefore, ſince in the Triangles P T Q and R V S the Sum of the Angles at the Baſe P Q is greater than their Sum at the Baſe R S, the vertical Angle R V S will be greater than the vertical Angle P T Q. Q. E. D.

LEMMA VIII.

If in three Lines b I, *g* I, *d* I *containing the equal Angles b* I *g, and g* I *d, three the leaſt refrangible Rays are incident at* I *upon the Surface* I K, *and are refracted from a rarer into a denſer Medium, whoſe refracted Rays drawn backwards let be* IB, I G, I D; *farther if of three the moſt refrangible Rays incident in the ſame Lines b* I, *g* I, *d* I, *their refracted ones are drawn backwards* I b, I *g*, *I* d; *the Difference of Refraction of the Rays, whoſe*

whose Incidence is the least, together with the Difference of Refraction of those, whose Incidence is the greatest, will be greater than twice the Difference of those, whose Incidence is in the Middle, that is Angle BIb + *Angle* DId > 2 *Angle* GIg.

FOR any Circle A D G being dedescribed touching the refracting Surface in I, whose Diameter let be A I, and which may cut the aforesaid Rays in the Points *b, g, d,* B, b, G, g, D, d. Conceive Subtenses to be drawn from A to these Points; and A*b,* A*g,* A*d* will be amongst one another, as are A B, A G, A D; and also as are A b, A g, A d: Whence it follows, that A B, A G, A D are amongst one another, as A b, A g, A d. And further, by *Lemma* VI. that the Arch G D is > Arch B G, and Arch g d > Arch bg. Now let the Arch G M be = B G, and it will be G D > G M and A D > A M. Also in the Circumference A D take any Point N on this Condition, that if you conceive the Subtenses A M, A N to

be drawn, it may be A B. A b :: A M. A N, and A B, A G, A M will be amongst themselves, as are A b, A g, A N; and consequently since the Arches B G and G M are equal, the Sum of the arches B b + M N (by *Lemma* VIII.) will be greater than the Double of the Arch G g. But since it is A M. A N (:: A B. A b) :: A D. A d, or conversly A M. A D :: M N. D d; on account of A D > A M, it will be Arch D d > Arch M N, and the Arch B b being added on both sides, it will be Arch B b + Arch D d > Arch B b + Arch M N, and much more it will be Arch B b + Arch D d > 2 Arch G g, or Angle B I b + Angle D I d > 2 Angle G I g. Q. E D.

PROP. XXVI.

Fig. 53. *Heterogeneal Rays being refracted by a Prism, the Difference of the Angles, which the incident Rays make with the emerging ones, will then become the least, when the Refractions on both Sides are equal.*

In the Prism ABC let CR be taken equal to CS, and RS be drawn, and also any other Line PQ, that is not parallel to RS, and conceive within the Prism Rays, proceeding each way in those Lines PQ and RS, to go out at the Points P, Q, R and S, and the greatest refrangible Rays to be refracted towards K, M, H and O, and the least refrangible towards I, L, G and N. I say, that of the Refractions made unequally at P and Q, the Differences taken together IPK + LQM are greater than HRG + NSO the Differences taken together of the Refractions made equally at R and S. For the Differences of the Incidences at P, Q and S are equal, as was shewn in the preceeding *Proposition*, and consequently by *Lem.* VIII. the Difference of the Refraction of the difform Rays at P, where is the greatest Incidence, together with the like Difference at Q, where is the least Incidence, exceeds the Double of the like Difference at S, where is the middle Incidence. That is, Angle IPK + Angle LQM $>$ 2 Angle NSO, or since NSO is equal

equal to Angle I P K + Angle L Q M > Angle N S O + Angle G R H. Q. E. D.

SCHOL. I have ſuppoſed indeed the Rays to go out of the Priſm on both Sides, but if they proceed from I and K through P and Q to L and M, and from G and H, through R and S towards N and O, the Poſitions of the Lines and Quantities of the Angles will not be thence changed, and therefore the ſaid Demonſtration will then alſo hold; and for the ſame Reaſon it will alſo hold, when the Rays diverging to the Priſm, become in the Priſm parallel, which is in like Manner to be underſtood of the Demonſtrations of the XXIV. and XXV. *Propoſitions*. Moreover, in any other Caſes whatever; where they diverge before Refraction, and converge after, or are incident parallel on the Priſm, they will not ever ſo much recede from a Paraliſm within the Priſm, but that the Angles or Differences of the Angles, which the incident Rays make with the emerging ones, may be nearly eſteemed

esteemed for the same, as if they were parallel within, so that the said *Propositions* extend altogether to all Cases.

PROP. XXVII.

If Lastly *Rays being refracted from a given Point* F *to a given Point* X, *through a Prism given in Position, there is required the Angles* D F E, G X H, *which the Heterogeneal Rays comprehend.* Fig. 54.

THE Problem is of the Number, that the *Ancients* called *Linear*, but the following mechanical Solution approaches the Truth, as much as practical Things require. Feign the Sum of the Angles D F E + G X H to be equal to the Angle N M O, made after a double Refraction by two Rays similar as to Refraction with the others F D and F E, and incident in the Line L M, nearly parallel to a right Line bisecting the Angle D F E. And of the Rays refracted to X produce some one, as G X meeting with the incident

Ray

Ray F D in V, to *f*, that *f* may be the Place of the Image, which the Object F exhibits to the Eye at X. Then the Angle O M N and the Distances *f* X and *f* V, being mechanically known, say as *f* X. *f* V : : Angle N M O. Angle G X H, and G X H will be very nearly what you seek. As is in some Part manifest, from what has been shewn at the *Scholium* of *Prop.* XII. When the Refractions on both Sides are not very unequal, the Business is more expeditiously done by the *Schol.* to the same *Proposition*, by making as V X. F V : : Angle D F E. Angle G X H, or by Composition F V + V X. F V : : Angle N M O. Angle G X H.

SECTION

SECTION IV.

Of the REFRACTIONS *of Curve Surfaces.*

THUS much of the Refractions of Planes; it is now time to treat of Curves and especially spherical Surfaces; the Doctrine whereof in respect of homogeneal Rays I shall endeavour to comprise in the following *Propositions.*

PROP. XXVIII.

A Ray being incident on a curve Surface to draw its refracted Ray.

THE Refraction of a Ray by a Curve is the same, as by a Plane touching the Curve in the Point of Refraction. Seek therefore the Ray refracted by the touching Plane by *Prop.* III.

PROP. XXIX.

If Rays, whether parallel or proceeding from or to a Point, fall upon a Sphere

Sphere to be refracted; to determine the Concourse of the refracted Rays nearest to the Axis or to find the Focus. †

Fig. 55. LET A be a Point sending out Rays towards a spherical Surface B N P, described to the Center C: From the Vertex and Center erect to the Axis A C the Perpendiculars B H and C I; and thro' the Point A draw any Line H I meeting them in H and I. Then from the Point C towards I, take C R, which may be to C I, as the Sine of Refraction to the Sine of Incidence; and draw the right Line H R meeting A C in Z; and Z will be the Concourse of the refracted Rays, which was to be determined. For let A N be a Ray falling the nearest to the Axis at N, and meeting C I in K. Draw N Z meeting C I in *g*, and as the manner is, conceive the infinitely small Arch B N to be equal to B M, a Segment of the right Line B H terminated at the Ray A K,

† This most elegant Way of finding the Focus in spherical Figures was first published by Dr. *Barrow*, at the End of his XIV[th] *Optical Lecture*, as communicated to him from our Author.

and

and it will be, C I. C R :: C K. C *g*: * That is, C K to C *g*, as is the Sine of Incidence to the Sine of Refraction. And therefore, since the Angles C A K and C Z *g* by Hypothesis are infinitely small, and consequently B N to K N, and C *g* to N Z Perpendiculars, or at least equipollent to Perpendiculars, N Z will be the refracted Ray of A N. Q. E. D.

COROL. 1. PUTTING I to R, as the Sine of Incidence to the Sine of Refraction; it will be $\frac{I}{R}$ A B. A C :: B Z. C Z. For it is $\frac{I}{R}$ A B. A B (:: I. R) :: C I. C R, and A B. A C :: B H. C I, and by perturbated Equality. $\frac{I}{R}$ A B. A C (:: B H. C R) :: B Z. C Z.

COROL. 2. IF the Point A be infinitely distant, or sends out parallel Rays, then by reason B H and C I are equal, it will be I. R :: B Z. C Z. And

* For C I. C K: (B H. B M::) C R. C *g*, and by Permutation, C I, C R :: C K. C *g*.

so

ſo if the refracted Rays are parallel, then by Reaſon B H and C R are equal, it will be I. R : : A C. A B.

COROL. 3. IF of the four Points A, B, C and Z, any three are given, the fourth may be found, as will appear by the following Examples.

EXAMP. 1. LET A, B, C be given, and Z be ſought. It is $\frac{I}{R}$ A B. A C : : B Z. C Z. And conſequently by Diviſion $\frac{I}{R}$ A B — A C. A C : : B C. C Z.

EXAMP. 2. IF A, B and Z be given, and C ſought; ſince it is $\frac{I}{R}$ A B. A C : : B Z. C Z, by Alternation it will be $\frac{I}{R}$ A B. B Z : : A C. C Z, and by Compoſition $\frac{I}{R}$ A B + B Z $\frac{I}{R}$ A B : : A Z. A C.

EXAMP. 3. IF A, C and Z be given, and B ſought; ſince it is $\frac{I}{R}$ A B.

A C :: B Z. C Z, or AB. $\frac{R}{I}$ AC :: B Z. C Z; it will be by Inverſion and Alternation $\frac{R}{I}$ A C. C Z :: A B. B Z, and by Compoſition $\frac{R}{I}$ A C + C Z. C Z :: A Z.B Z.

THE ſame Things may be determined by the drawing of Lines; as if A, B, and Q being given, C was ſought. Erect B H of any Length perpendicular to A Z, and in it take B G, which may be to B H, as I to R, join A H and G Z meeting in I, and I C, being let fall perpendicular on A Z, will fall at the Point ſought C.

NOTE 1. That Z is the Place of the Image of the Object A exhibited by Refraction, when the Spectator's Eye is placed in the Axis beyond Z.

2. IF the refracted Rays diverge, or the incident Rays converge, or are parallel, the Conſtruction of the *Problem* will be the ſame, thoſe Things being only changed, that ought to be changed.

3. IF the Rays, emitted from the Point A, are transmitted successively thro' several spherical Surfaces having the same Axis AC; to determine the Concourse after all the Refractions, seek first the Concourse of the Rays after the first Refraction; then the Concourse of the same after the second Refraction, as if they had been primarily emitted from the Point of the preceding Concourse; and so on till you are arrived at the last Concourse. And by this means the Place of the Image of any Object seen through a Telescope or Microscope may be determined.

4. BY Help of *Corol.* 3. convex Glasses may be made of spherical Surfaces, which may serve for making Telescopes after any designed manner. For it appears from that *Corol.* that not only the Refractions of given convex Glasses may be investigated, but also the Glasses be delineated, which shall produce given Refractions.

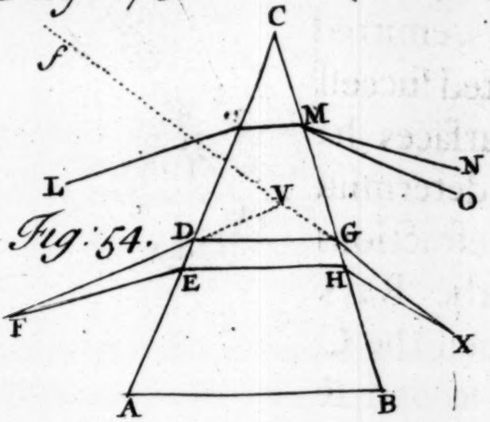
LEM-

Pl

F

F

K



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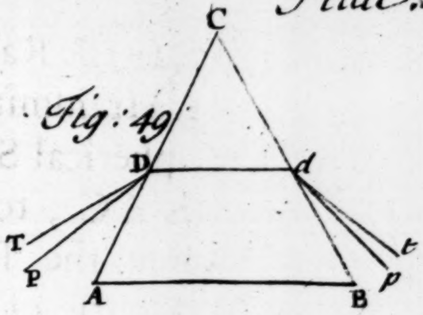


Fig: 49

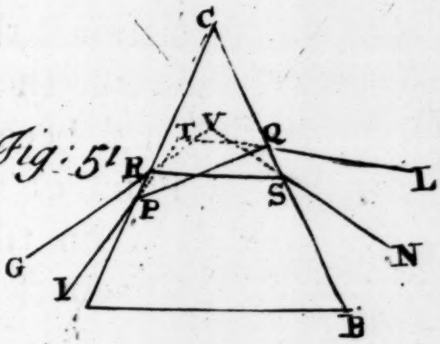


Fig: 51

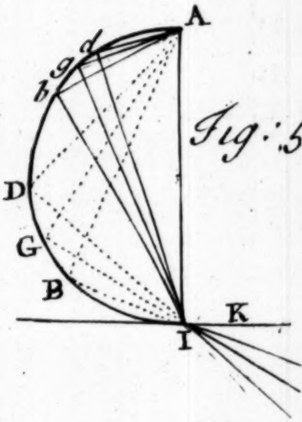
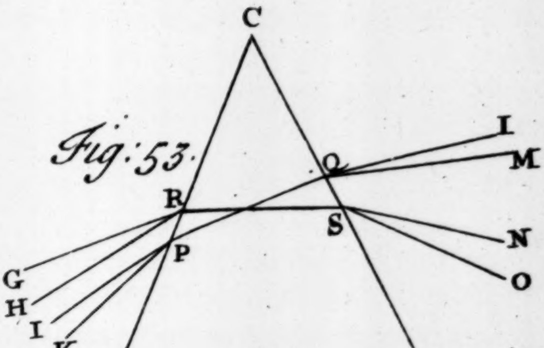


Fig: 50.



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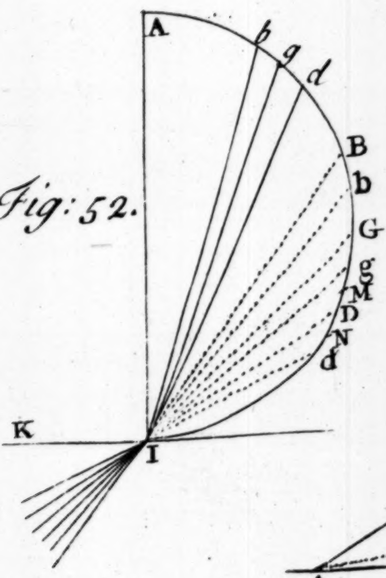

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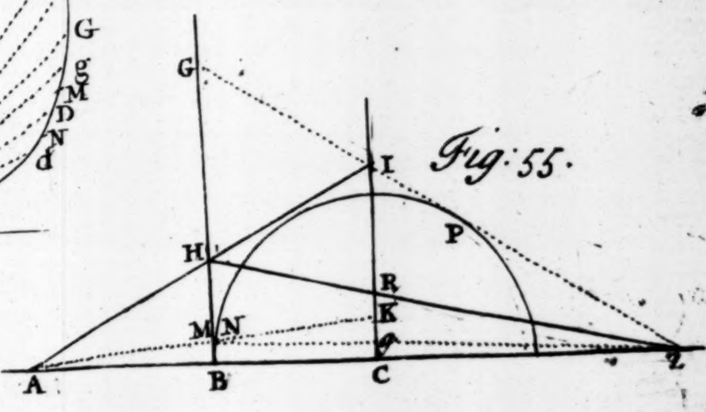

Fig: 55.

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LEMMA IX.

In any given Curve, to determine the Concourse of the Axis and the nearest Perpendicular.

IN *Fig.* 56. Let B N n be a Curve, and to any of its Points n indifferently taken, seek a Perpendicular $n\ c$ by the known Method of drawing Perpendiculars to Curves, and at the same Time you will find the Length B c. Then ($n\ t$ being let fall perpendicular to B c) feign B t or $n\ t$ to be infinitely small, or nothing, and there will come out the Length B C, whose Termination is at the Concourse of the Axis with the nearest Perpendicular. *Fig.* 56.

EXAMP. 1. Let B N n be a Parabola, whose *Latus rectum* let be r, and B t call x, it will be B $c = x + \frac{1}{2} r$, as is known. Now put $x = 0$, and there will remain $\frac{1}{2} r$ for the Length B C at the Vertex.

EXAMP. 2. Let B N n be an Ellipsis whose *Latus rectum* let be r and *Latus transversum* q; it will be, as is known, B $c = x + \frac{rx}{q} + \frac{1}{2}r$. Now put $x = 0$, and there will remain again $\frac{1}{2}r$ for the Length B C at the Vertex. Nor is the Process different in more compounded Curves.

PROP. XXX.

Rays being incident very nearly perpendicularly on any curve Surface; to determine the Concourse of the refracted Rays or their Focus.

Fig. 57. LET P B Q (in *Fig.* 57.) be any Curve. A the common Point or the Concourse of the incident Rays, A B a perpendicular Ray or the Axis, and A N a Ray very nearly perpendicular, or the nearest to the Axis. And let N C be perpendicular to the Curve, and meeting the Axis A B at C. And the Point C being found by *Lem.* IX. erect at B and C the Perpendiculars B H and C I,

C I, draw any Line A I meeting them in H and I, towards I take C R, which may be to C I, as the Sine of Refraction to the Sine of Incidence; and the right Line H R will meet A B in Z the sought Concourse of the refracted Rays.

IT is proved after the manner of the preceding *Proposition*, and to this belong the like *Corollaries* and *Notes*.

PROP. XXXI.

Parallel Rays being incident on a Sphere; to determine the Error from the principal Focus of Rays remote from the Axis.

IN *Scheme* 58. let N B *n* be a Sphere, C its Center, C B a Semidiameter parallel to the incident Rays, A N an incident Ray, and N K its refracted Ray meeting the Axis, or the Semidiameter C B in K, and F being supposed the principal Focus, that is, into which the Rays lying near the Axis are collected, there is to be sought the Error F K. Let fall therefore the Perpendiculars C E on *Fig.* 58.

N K and N G on C K, and make $CB = a$, $GB = x$ and $CK = z$; and from the Nature of the Circle it will be $NGq = 2ax - xx$, to which add GKq, that is, $zz + 2xz - 2az + xx - 2ax + aa$, and there will come out $NKq = zz + 2xz - 2az + aa$. Now since N G is to C E, as the Sine of Incidence to the Sine of Refraction, or as I to R, and on account of the similar Triangles C E K and N G K, N K and C K are in the same Ratio; it will be $II.\ RR\ (::NKq.\ CKq) :: zz + 2xz - 2az + aa.\ zz$: and therefore $II \times zz = RR \times zz + 2RR \times xz - 2RR \times az + RR \times aa$. And by Reduction of the Equation,
$$zz = \frac{2RR \times az - 2RR \times xz - RRaa}{RR - II},$$
and the Root being extracted
$$z = \frac{RRa - RRx + R\sqrt{IIaa - 2RRax + RRxx}}{RR - II},$$
i. e. (the radical Quantity being thrown into * an infinite *Series*) $z = \frac{Ra}{R - I} -$

* The *Method of Series* was a very early Invention of our Author's, *viz.* in 1665. A particular Account of which may be seen in a Book called *Commercium Epistolicum Johannis Collins & aliorum de Analysi promota*, first printed at *London* in 4to. *A.* 1712, and a second Edition with Additions in 8vo. *A.* 1722.

RR

$\frac{RR \times x}{IR - II} - \frac{R^3 \times xx}{2I^3 \times a} - \frac{R^5 \times x^3}{2I^5 \times a^2}$ &c. Now since by *Corol.* 2 or 3. to *Prop.* XXIX. $\frac{R \quad a}{R - I}$ is $=$ CF (which may be known from the Value of z now found, by feigning $x = 0$) from this CF take the found Value of z and there will remain $\frac{RR \times x}{IR - I9} + \frac{R^3 \times x^2}{2I^3 \times a}$, &c. for the Value of the Error KF, which we sought.

COROL. 1. If BG or x be supposed very small, $\frac{RRx}{IR - II}$ will be very nearly equal to KF; for then the Quantities $\frac{R^3 \times x^2}{2I^3 \times a} + \frac{R^5 \times x^3}{2I^5 \times a^2}$ &c. on Account of the ascending Powers of the same x, will become very small, and in Respect of the Term $\frac{RRx}{IR - II}$ may be looked on as nothing.

COROL. 2. MOREOVER if you make NG $= y$, it will be $\frac{RRyy}{2IRa - 2IIa} =$ KF nearly, for it is NG $=$ BG $\times$ $\overline{BC + CG}$ or $=$ BG $\times$ 2BC nearly,

nearly, that is, $yy = 2ax$ nearly, or $\frac{yy}{2a} = x$, and $\frac{yy}{2a}$ being substituted for x in the Value of KF, $\frac{RRyy}{2IRa - 2IIa} =$ KF.

COROL. 3. HENCE the Errors KF are as the versed Sines GB, or as the Squares of the half Chords NG.

COROL. 4. IF the Ray ANK be given in Position, and the refracted Ray nk of any parallel Ray nearer the Axis, and falling on the other Side of the Axis, be drawn cutting the Axis in k, and the refracted Ray NK in Q, and to the Axis be let fall the perpendicular Qo: the Line Ko will become the greatest of all, when the Ray an is nearly twice less distant from the Axis than the other Ray AN. For gn being let fall perpendicular to the Axis, put $ng = v$, Ko $= S$, GK $= F$ and KF $= h$, and by *Corol.* 3. of this it will be $\frac{hyy - hvv}{yy} =$ Kk, farther it is GK, GN :: Ko. Qo, and

and consequently $Q\,o = \frac{yS}{F}$; also gn, GK ($= gk$ nearly) :: $Q\,o$. ok, wherefore $ok = \frac{yS}{v}$. To this add $K\,o$, and there will again come out $Kk = \frac{vS + yS}{v}$. Wherefore it is $\frac{vS + yS}{v} = \frac{hyy - hvv}{yy}$, and Division being made by $v + y$, and the Equation reduced, there comes out $S = \frac{hvy - hvv}{yy}$.

Now that the greatest S may be found, multiply the Terms according to * *Hudden*'s *Method*, by the Dimensions of the indeterminate Quantity v, and there will come out $o = \frac{hvy - 2hvv}{yy}$, or $y = 2v$, that is $NG = 2\,ng$.

* Here our Author refers to *Hudden*'s Method *de Maximis et Minimis* printed *Anno* 1659 in *Cartes*'s *Geometry*; because his own *Method of Fluxions* was not yet made publick, though he had written several small Tracts on this Subject in 1665, before he read these Lectures, and a larger one in 1671. These have been never yet printed, though many Copies of them in Manuscript are got abroad. The last is frequently mentioned in the *Commercium Epistolicum*.

COROL.

COROL. 5. AND hence K o, when it is greateſt, is equal nearly to a fourth Part of K F; for in the now found Value of S, if you write $2v$ for y, there will ariſe $\frac{h}{4} = S$.

COROL. 6. IT is alſo $oQ = \frac{Ry^3}{8Iaa}$. For it is I K ($=$ B F nearly). G N :: K o. o Q, that is $\frac{Ra}{RI-Iy}$. y :: $\frac{RRyy}{8IRa-8IIa}$ ($= \frac{1}{4}$ K F). $\frac{Ry^3}{8Iaa}$.

COROL. 7. IF the Arch B M be taken equal to B N and B m = B n, and the Rays refracted at the Points M and m be drawn meeting one another in P. It is manifeſt that the Space P Q is $= \frac{Ry^3}{4Iaa}$, *viz.* double of o Q; and it is farther manifeſt, that the refracted Rays of all the Rays falling upon the ſpherical Surface between N and M will converge into this Space P Q, and that the ſame Space P Q is the leaſt circular Space, into which all the Rays may be collected; and con-

consequently is the Focus or Place of the Image of the Object sending parallel Rays to the Lens, whose Aperture is the Limits M and N. *viz.* no Rays can pass beyond this Space, because since *o* Q is in a given Ratio to K *o*, and *o* Q will be at the same Time the greatest, and consequently the Point Q is the remotest from the Axis of all the Points lying towards F, in which any Ray meets with the external Ray N K; nor can they be collected into a less Space, because the Rays N K and M K cut the external Rays in the Points P and Q, by which the Space P Q is terminated.

COROL. 8. IF N B M the Aperture of the Circle be enlarged, or diminished, the lateral Error P Q will be as y^3 or as the Cube of the Breadth of the Aperture N M. Also if the Aperture remaining unaltered, the Magnitude of the Circle be changed, the Error P Q will be reciprocally as *a a*, or as C B q; and consequently as B F q: For C B and B F are in a given Ratio. But if both the Magnitude of the Circle

Circle and the Aperture be altered; that Error P Q will be as $\frac{y^3}{aa}$ or as $\frac{\text{N M cub.}}{\text{B F q}}$, as from $\frac{\text{R}\, y^3}{4\, \text{I}\, aa}$ the Value of P Q may be manifest.

SCHOL. NEARLY after the same Manner, as we have determined the Errors C F and P Q of parallel incident Rays; may be determined the like Errors of diverging or converging Rays, but with a more difficult Calculation.

PROP. XXXII.

If Rays, whether parallel, or inclined towards some common Point, are opposed to a Sphere to be refracted; to determine the Concourse of the refracted Rays lying out of the Axis, the nearest to one another, and in the same Plane with the incident Rays.

Fig. 59. IN *Fig.* 59. Let A N be an incident Ray, N K its refracted one, and N V a right Line in the Plane of the Tri-

Triangle A N K touching the Sphere at N. To A N draw N R perpendicular, and meeting the Axis A C in R; as alſo R V parallel, and meeting the Tangent N V in V. Alſo to N K draw N Q perpendicular and V Q parallel meeting in Q, and draw Q C meeting N K in Z; and Z will be the Concourſe of the Rays the neareſt to A N. For let A *n* be another of the incident Rays infinitely near to the former A N, and meeting N R in G. Draw *n* Z meeting N Q in H; and to A N and N K from C the Center of the Sphere let fall the Perpendiculars C D and C E meeting A *n* and *n* Z in *d* and *e*. Now ſince A *n* is ſuppoſed infinitely near to A N, the infinitely ſmall Arch N *n* may be looked on as a right Line coinciding with the Tangent N V, and the Triangles N G *n*, N R V; and N H *n*, N Q V as ſimilar. Wherefore it is D C. D *d* (: : N R. N G : : N V. N *n* : : N Q. N H) : : E C. E *e*. Whence by Converſion and Alternation D C. E C : : *d* C. *e* C. But it is D C to E C, as the Sine of Incidence to the Sine of Refraction, becauſe that N K is the refracted

fracted Ray of A N; and consequently also *d* C to *e* C is, as the Sine of Incidence to the Sine of Refraction. And therefore since the Angles D A *d* and E Z *e* are infinitely small, and for that Reason C *d* to A *n* and C *e* to *n* Z Perpendiculars, or at least equipollent to Perpendiculars, *n* Z will be the refracted Ray of A *n*. Q. E. D.

COROL. 1. It is N D. N E (or NP. N F) : : N R. N Q. For N C being drawn, on account of the similar Triangles N R V and N D C; N E C and N Q V, it is N D. N R (: : N C. N V) : : N E. N Q, and by Permutation N D. N E. : : N R. N Q.

Hence is derived a more ready Resolution of the *Problem*, viz. to the Rays A N, N K erect the Perpendiculars N R, N Q, whereof N R may meet the Axis A C; and N Q be to N R, as N F to N P. Then draw Q C, which will meet with N K in the sought Point Z *.

* This most elegant Construction was published as from our Author by Dr. *Barrow* in his XIIIth *Optical Lecture Art.* XXVI.

COROL. 2. IT is alſo A N x D C x N E. A D x E C x N D : : N Z. E Z. For it is A D. A N : : D C. N R, and thence $N R = \frac{AN \times DC}{AD}$. Alſo N D. N E : : N R. N Q, and thence $N Q = \frac{AN \times DC \times NE}{AD \times ND}$; and conſequently A N x D C x N E. A D x N D x E C (: : N Q. E C) : : N Z. E Z.

COROL. 3. IF the radiating Point A is infinitely diſtant, or emits parallel Rays, putting I. R : : Sine Incidence. Sine Refraction, it will be I x N F. R x N P : : N Z. E Z, For in this Caſe A N and A D ſeeing they are infinitely long, ought to be looked upon as equal; and conſequently by *Corol.* 2. of this, it will be D C x N E. E C x N D : : N Z. E Z. But by Hypotheſis it is D C. E C : : I. R; and therefore I x N E. R x N D(: : N Z. E Z) : : I x N F. R x N P. But of theſe ſee more in Dr. *Barrow's Lectures*.

BUT

BUT there may be obſerved 1. That by making the proper Changes, the Reſolution of the *Problem* may be eaſily accommodated to any Caſe whatever; whether the incident Rays diverge from any Point, or converge to the ſame, or fall parallel.

2. SINCE of the Rays neareſt to A N K, thoſe that lie in the Plane A N R do meet in Z, but thoſe, that lie in a conical Surface generated by the Revolution of the Triangle A N K about its Side A K, do meet in K, the greateſt Conſtipation of the Rays neareſt on all Sides to A N K will be about the Middle of the Space K Z, as at Y; and therefore the Eye being placed in the Line N K beyond K, the Place of the viſible Image of the Object A, ſeen by the Refraction of the ſpherical Surface B N, will be at Y, or at leaſt within the Limits K and Z; for that Place is not preciſely determined.

3. When the Rays are ſucceſſively refracted by ſeveral Surfaces, that you may determine the Concourſe of the near Rays after all the Refractions, firſt ſeek the Concourſe after the firſt Refraction, then the Concourſe of the ſame after the ſecond Refraction, as if they had flowed primarily from the Point of the preceding Concourſe; and ſo on, as it was ſaid at *Prop.* XXIX.

PROP. XXXIII.

Rays being incident upon any curve Surface whatever, to determine the Concourſe of the refracted Rays neareſt to one another, and lying in the ſame Plane with the incident ones.

In *Fig.* 59. feign B N P now to repreſent not a Sphere, but any other Curve, and let A be the common Point, or Concourſe of the incident Rays, A N one of the incident Rays, N K its refracted one, and N C a perpendicular to the Curve at the refracting Point. In this N C ſeek the Interſection of any *Fig.* 59.

the nearest Perpendicular (such as *n* C) insisting on another the nearest refracting Point. Which thing shall be taught † elsewhere. And let that Intersection be C; now A C being drawn, let fall to the Rays A N, N E, the Perpendiculars C D, C E, and erect N R, N Q, whereof let N R meet A C in R, and let it be N Q to N R, as N E to N D, and Q C being drawn will meet with the refracted Ray N K in Z the desired Concourse of the nearest refracted Rays. It is proved after the manner of the preceding *Proposition*, and to this belong also the like *Corollaries* and *Notes*.

PROP. XXXIV.

To determine the Figure which shall so refract homogeneal Rays, whether parallel, or terminated at some common Point, that all the refracted Rays may meet accurately in some other given Point.

† *Viz.* in his *Treatises of Fluxions* mentioned before.

IN *Fig.* 60. let A be the Concourſe of the incident Rays, and Z of the refracted Rays, and let any Point B be taken at Pleaſure in the right Line A Z for the Vertex of the Curve. From that Point B in the Line B Z towards the denſer Medium let be taken B G of any Length, and B R in the Proportion to B G, that the Sine of Incidence has to the Sine of Refraction. And with the Centers A and Z, and the Intervals A G and Z R let be deſcribed Circles interſecting one another in N, and the *Locus* or Place of N will be the Curve, that will perform the deſired Refraction. *Fig.* 60.

THAT this may appear, let A N be produced to S, that it may be N S. N Z : : BG. B R ; and to N S and N Z let be erected the Perpendiculars S T and Z T meeting in T, and N T being drawn will touch the Curve in N, as will be manifeſt from the *Method of drawing Tangents* elſewhere * explained. Now ſince N S and N Z are as B

* In our Author's *Treatiſes of Fluxions.*

G and B R, that is, as the Sine of Incidence and Refraction; and in respect of the whole Sine or Semidiameter N T, N S is the Sine of the Angle N T S, which is equal to the Angle of Incidence of the Ray A N, and N Z the Sine of the Angle N T Z, which is equal to the Angle of Refraction of the Ray N Z; it is manifest that N Z is the refracted Ray of A N. Q. E. D.

NOTE 1. A Curve may also be described for this Purpose, which shall pass through any given Point B placed out of the Axis A Z. *Viz.* in *Fig.* 61. let be drawn A B and Z B, and in them let be taken B G to B R as I to R, and let Circles be described meeting in N, and N will be in the Curve, that was to be described.

Fig. 61.

2. THE said Resolution of the *Problem* by making necessary Changes, extends it self to all Cases, whether the incident or refracted Rays converge, diverge, or are parallel, or whether the the Refraction is made out of a rarer Medium into a denser or out of a den-

ser

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Plate

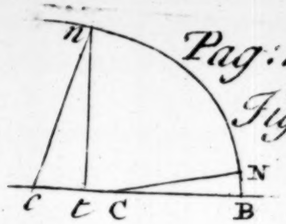


Fig:56

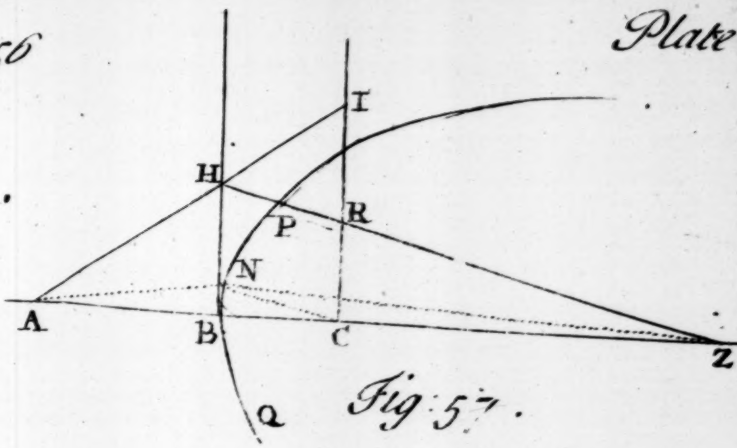


Fig:57.

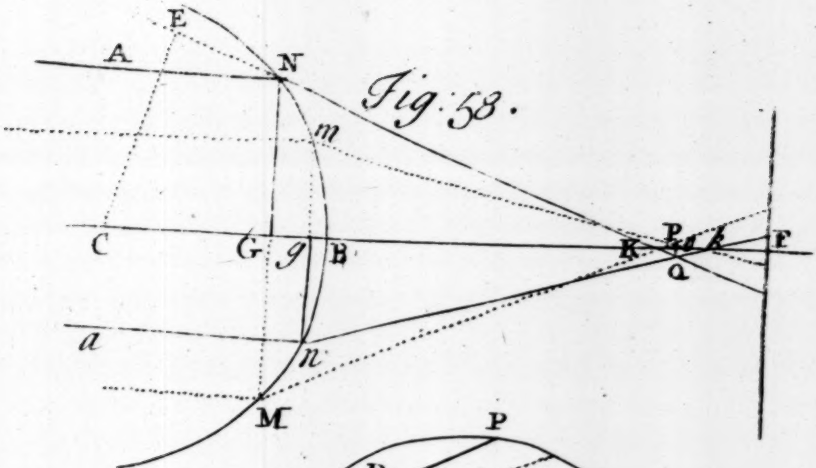


Fig:58.

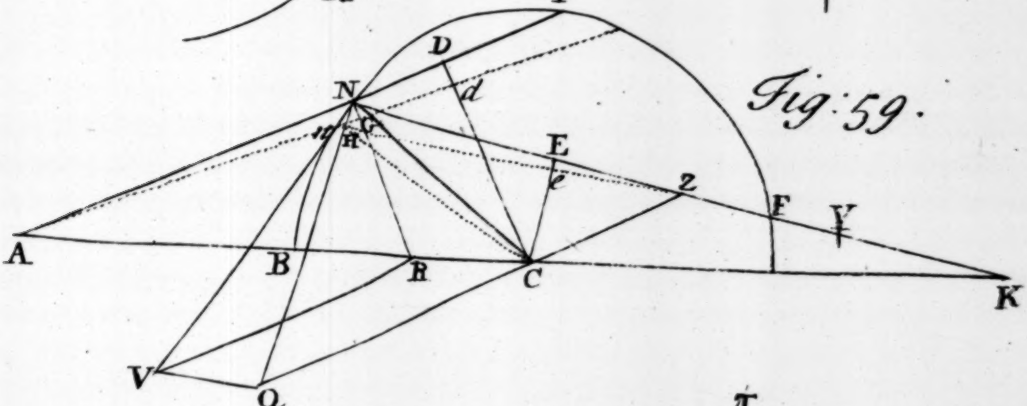


Fig:59.

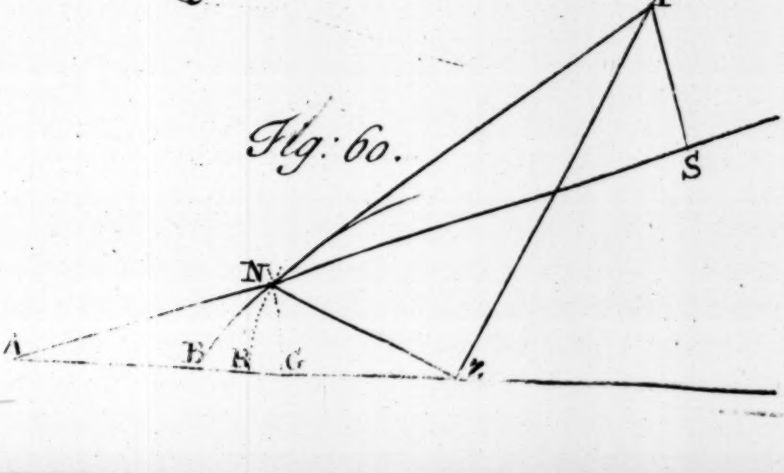


Fig:60.

ſer into a rarer. And indeed if the Rays are on no Side parallel, that is, if neither of the Points A and Z is at an infinite Diſtance, the Curve will be one of the four *Ovals*, which *Cartes* has deſcribed for this End in his *Geometry;* but if either of them is infinitely diſtant, ſo that the Rays reſpecting that Point become parallel, the Curve will be a conick Section, as is known. And in this Caſe the Circle R N or G N on account of the infinite Diſtance of the Center will become a right Line perpendicular to A Z at R or G. *

LEMMA X.

Of parallel Rays refracted by a Circle, to determine that Ray, whoſe Part included within the Circle may have a given Ratio to the Part of its refracted Ray included within the ſame Circle.

IN *Fig.* 62. Let A N be the incident Ray, N K the refracted one, N P and *Fig.* 62.

* See his *Principia Prop.* 97, *Lib.* 1.

N F their Parts included within the Circle, C D and C E Perpendiculars let fall to those Parts from the Center of the Circle, and B C a Semidiameter drawn parallel to A N. And let it be C D. C E : : I. R, and N P. N F : : *p*. *q*. These Things being supposed, that there may be known the Point N, which determines the Rays A N and N K, erect at B C a Perpendicular B X, whose Square let be to the Square of B C, as $\frac{qq-pp}{pp}$ to $\frac{II-RR}{II}$; and C X being drawn will cut the Circle in the sought Point N. For it is by Hypothesis *p*. *q* (: : N P. N F) : : N D. N E, and I. R : : C D, C E; wherefore $\frac{q}{p}$ N D $=$ N E, and $\frac{R}{I}$C D $=$ C E. Moreover since it is N D q $+$ C D q ($=$ N C q) $=$ N E q $+$ C E q, take from both Sides N D q $+$ C E q, and there will remain C D q $-$ C E q $=$ N E q $-$ N D q, that is, by substituting the Values of C E and N E just now found, C D q $-$ $\frac{RR}{II}$ C D q $=$ $\frac{qq}{pp}$ N D q $-$ N D q, and Reduction being made,

$$\frac{II - RR}{II} \, CDq = \frac{qq - pp}{pp} \, NDq.$$

Which being resolved into a Proportionality becomes $\frac{qq-pp}{pp} . \frac{II-RR}{II}$ (: : C Dq. N Dq) : : B X q. B C q. Q. E. D.

PROP. XXXV.

The Sun shining upon a transparent Sphere, to determine the greatest Inclination of the Sun's Rays to the Axis, as they emerge after one Reflexion.

IN *Fig.* 63. Let B N K be the proposed Sphere; B Q a Diameter or Axis parallel to the incident Rays, A N any one of the incident Rays, N F its refracted one, F G the reflected one, and G R the refracted one again; and there is required the greatest Angle, which R G can make with the Axis B Q. For which Purpose it must be observed, that in the Case only, when R G are inclined the most to B Q, the Rays the nearest to A N can emerge parallel to R G. For in other Cases, of emerging Rays the nearest to one another, some are continually more incli- *Fig.* 63.

ned, others leſs to BQ; and conſequently are ſomewhat inclined to one another. It muſt be farther obſerved, that thoſe Rays emerge parallel, which meet at the Point of Reflexion. For draw the Ray *a n* parallel and the neareſt to A N, and let its refracted Ray be *n f*, reflected *f g*, and refracted again *g r*; and the Points F and *f* coinciding, ſince the Angles N F *n* and G F *g* are equal, and the Refractions at N *n* and G *g* alike, the emerging Rays GR and *g r* will be equally parallel as the incident Rays A N and *a n*.

THERE is therefore to be ſought a Ray A N, whoſe refracted Ray meets at F with the refracted one of the neareſt Ray *a n*; and indeed by *Corol.* 3. *Prop.* XXXII. (C D and C E being let fall Perpendiculars from the Sphere's Center to the Rays, and it being made I. R : : C D. C E) if the Rays themſelves concur at any Point Z, it will be I × N F. R × N P : : N Z. E Z : : N F. E F (*viz.* the Point Z falling by Hypotheſis on F) : : 2. 1. Wherefore I × N F = 2 R × N P, and I. 2 R : : N P N F.

N F. There is therefore given the Ratio of N P to N F; and thence, by *Lemma* X. there will be given the Point N. Thus to the Vertex of the Circle let be drawn the Tangent B X, whoſe Square may be to the Square of the Semidiameter B C, as 4 R R — I I to I I — R R, and let C X be drawn; for this will meet the Circle in N; and from N being found, the reſt are determined with no Trouble.

COROL. 1. HENCE it becomes 3 R R. I I — R R :: C N q. N D q. For ſince it is 4 R R — II. I I — R R :: B X q. B C q; it will be by compounding 3 R R. I I — R R (:: B X q + B C q = C X q. B C q) C N q . N D q .

COROL. 2. IT is alſo I. 2 R :: N D. N E. For above it was I. 2 R :: N P. N F. And from theſe the Solution of the *Problem* becomes the more expeditious.

SCHOL. TOGETHER with the greateſt Inclination of the Ray R G, there is given the greateſt of the Arches F Q is

is terminated at the refracted Rays N F. For the Angle F C Q, which F Q ſubtends, is equal to the Angle, which C F and A N comprehend; that is, equal to Half the Angle, which R G and A N or B Q comprehend; and therefore of the Arches F Q, as well as of the Angles comprehended by R G and B Q, that is the greateſt, which is defined by the Ray A N incident at the Point now found.

PROP. XXXVI.

The Sun ſhining upon a Tranſparent Sphere, to determine the leaſt Inclination of the Rays to the Axis after two Reflections.

LET A N and *a n* be two incident Rays the neareſt to one another, which after two Reflections in F *f*, and G *g*, let them emerge in H S and *h s*, and it is manifeſt, that in that Caſe only, where the acute Angle, which B Q and S H comprehend, is the leaſt, thoſe Rays H S and *h s* can be parallel, as was ſaid above of the Rays G R and

g r;

gr; and where this happens, the Ray F G alſo will be parallel to fg. Whence 2 arc Ff (= arc Ff + G g = arc F G — fg = arc N F — nf) = arc Nn — Ff. And conſequently 3 arc Ff = arc Nn; and ſince N F is divided in Z in the Ratio of theſe Arches, as is manifeſt; it will be N Z = 3 Z F or 3 E Z. Since therefore by *Corol*, 3. *Prop*. XXXII, it is I × N F . R × N P :: N Z. E Z. or :: 3. 1; it will be I × N F = 3 R × N P. or I. 3 R :: N P. NF. There is therefore given the Ratio of N P to N F; and thence by *Lemma* X. there will be given the Point N, *viz.* by drawing B X, that ſhall touch the Circle in the Vertex B, and whoſe Square may be to B C *quad.* as 9 R R — I I to I I — R R; and by drawing C X, that ſhall meet the Circumference in N; but N being found, the reſt are eaſily determined.

COROL. 1. HENCE it is 8 R R . I I — R R :: C N q. N D q. For 9 R R — I I. I I — R R :: B X q. B C 9, and by compounding 8 R R . I I — R R (:: C X q. B C q) :: C N q. N D q.

COROL. 2.

COROL. 2. IT is also I. 3 R :: N D. N E; seeing it was before I, 3 R :: N . P . N F.

SCHOL. AFTER the same Manner will be found the greatest Inclination to the Axis of the Ray K T emerging after three Reflections, as well as the greatest of the Arches Q G . *viz.* in that Case F G and *f g* will meet in G, and it will be the Arch F *f* (= Arch F *g* — *f g* = Arch N F — *n f*) = N *n* — F *f*, and thence 2 Arch F *f* = Arch N *n*, and N Z = 2 Z F. And consequently 4. 1 :: N Z. E Z :: (*per Corol.* 3. *ad Prop.* XXXII.) I x N F . R x N P, or I . 4 R :: N P. N F; and therefore by *Lem.* X. 16 R R — I I, I I — R R :: B X q. B C q. Whence it follows, that 15 R R . I I — R R :: C N q. N D q . And I. 4 R :: N D. N E.

AND so if there be required the least Inclination of a Ray emerging after four Reflections, you will determine it by making 25 R R — II . I I — R R :: B X q . B C q, or 24 R R . II — R R ::

R :: CNq . NDq, and I . 5R :: ND . NE. And so on *in infinitum*. *

THE Refractions of homogeneal Rays having been handled, it now remains, that we come to heterogeneal ones. Of the Refractions of these in regard to Planes, we have treated the more diffusely, that thereby the Affections of Prisms (whose Use in making Experiments will hereafter be very frequent) might be known. But the principal Thing about Curve Surfaces, that now occurs to be determined, is the Quantity of the Error of the Rays, from whence arises the Confusion, or indistinct Vision of Objects, which in Telescopes is wont to happen through the too large Aperture of the object Glass. And to this End, since there was premised *Prop.* XXXI. Whence the Errors are known, that are made in spherical Surfaces through the Unfitness of the Figure: We shall now sub-

* The Use of the two last *Propositions* is to determine the Rain-bow, as see our Author's *Opticks*, *Book* I. *Part* II. *Prop*. IX.

join, how may be determined the Errors arising from the unequal Refrangibility of the Rays.

PROP. XXXVII.

Heterogeneal Rays falling on a Sphere, to determine the Errors occasioned by the unequal Refractions of alike incident Rays.

Fig. 64. FROM the Point A (*Fig.* 64.) upon the Sphere N B M, described to the Center C, let two the greatest difform Rays be incident in any Line A N, whose refracted Rays are N F and N f, meeting the Axis in F and f, and on them let fall the perpendiculars C I, C P and C T. Now if an accurate Solution was required, the Refractions of the Rays N F and N f must be separately computed. But since the Arch N M is supposed to be a very small Portion of a Circle; we shall attain to the Truth very nearly by assuming the Angles C N I, C N P and C N T to be almost as their Sines. Let therefore I be the common Sine of Incidence,

dence, P the Sine of Refraction of the greatest refrangible Rays, and T the Sine of the least refrangible ones; and it will be Angle C N I. Angle C N P :: I . P, and Angle C N P. Angle C N T :: P. T, and by Division, Angle I N P. Angle C N P :: P — I, P, and Angle C N P. Angle P N T :: P. P — T; and by Equality Angle I N P. Angle P N T :: P — I. P — T.

TAKE now the Arch B M equal to the Arch B N, and of the Rays incident in A M draw the refracted ones M F, M *f*, meeting the former in V and X: Draw V X, and produce it, till it meets the incident Rays at G and H; and it is manifest, that V X is the Breadth of the least Space, into which all the Rays may be collected. And it is G X. V X (:: Angle G N X . Angle V N X nearly :: Angle I N P. Angle P N T) :: P — I . P — T; and G H + V X (2 G X). V H :: 2 P — 2 I . P — T, and by Division G H, V X :: P + T — 2 I . P — T. Whence P, T and I being given; there

there will be given the Ratio of G H to V X.

FOR Example, since I have above determined, that in Glass terminated by Air, it is I. P :: $44\frac{1}{2}$. $69\frac{1}{2}$, and I . T :: $44\frac{1}{2}$. $68\frac{1}{2}$; if it be assumed $I = 44\frac{1}{2}$, it will be $P = 69\frac{1}{2}$, and $T = 68\frac{1}{2}$, and $P + T - 2I = 49$, and $P - T = 1$: and consequently GH. V X :: 49. 1. nearly.

SCHOL. BY the Help of this and *Prop.* XXXI, the Errors of homogeneal Rays, which happen in spherical Surfaces, through the Unfitness of the Figure, may be compared with the Errors of heterogeneal Rays and it will appear, that these are far greater in small Portions of Spheres: and consquently, that the Heterogenity of the Light, and not the the Unfitness of the spherical Figure is the Cause, that Telescopes are not yet arrived to a greater Degree of Perfection.

Fig. 58, 64. LET us conceive for Example, that N M B. in *Fig.* 58. and 64, represents

ſents the Object Glaſs of a Teleſcope, whoſe anterior Surface N M let be plane, that it may only refract the Rays on its poſterior or ſpherical Surface N B M; and let us ſuppoſe C B the Semidiameter of this Sphere to be 10 Feet, that it may make a Teleſcope of near 20 Feet (or 240 Inches) long. And let its Aperture be two Inches, as the greateſt that can be uſed, with ſufficient diſtinctneſs of Viſion in theſe Sorts of Teleſcopes, that magnify the Object about 70 or 80 Times, and let the Sine of Incidence be to the Sine of Refraction, in the Confine of Glaſs and Air; as 11 to 17 nearly, as we have determined above. Theſe Things being ſuppoſed, there muſt be written 120 for a, 1 for y, 11 for I, and 17 for R in the Value of P Q, which we exhibited in *Corol.* 7. *Prop.* XXXI.

That is in the Term $\frac{Ry^3}{4Iaa}$ and there emerges $\frac{17 \text{ Inches.}}{4 \times 11 \times 120 \times 120}$ or $\frac{17 \text{ Inches}}{633\ 600} = $ P Q.

And this is the lateral Error of the homogeneal Rays ariſing from the Unfitneſs of the Spherical Figure. Farther

ther let us conceive the Rays A N and A M in *Fig.* 64. to be parallel, and the Aperture N M will be = 2 Inches = G H, which is to V X as 49 to 1, by the preceding. That is, V X = $\frac{2}{49}$ Inches, or the Error, which arises from the Separation of the heterogeneal Rays from one another, in the same Place of Concourse, will be $\frac{2}{49}$ Inches. Now, confer these Errors, and it will appear, that V X is to P Q (or $\frac{2}{49}$ to $\frac{17}{633600}$) as 1267200 to 833, or as 1521 to 1 nearly.

AND consequently that V X is above a Thousand and five Hundred Times greater than P Q; so great a Disproportion indeed, that P Q in Respect of V X may be looked upon as nothing. The Error V X since it is $\frac{2}{49}$ Inch is so great, that I wonder, how Objects are seen so distinctly through these Sorts of Telescopes. But the Error of the other Kind P Q or $\frac{17}{633600}$ Inch that is $\frac{1}{37271}$ Inch nearly, is by far too little to become

Fig: 61.

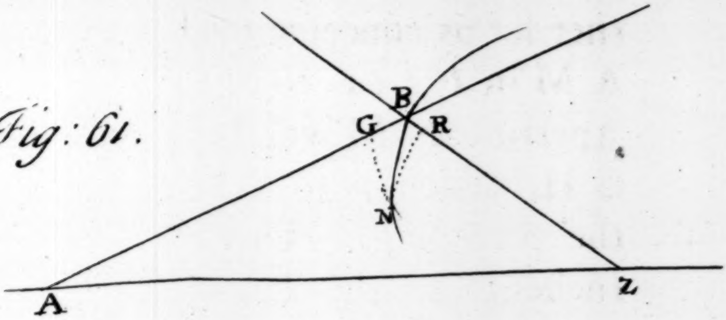


Fig: 63

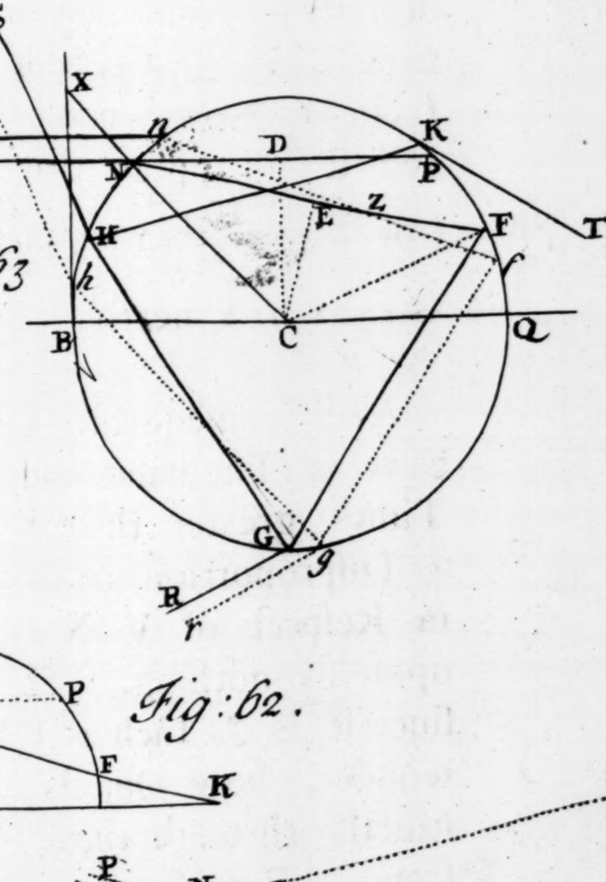
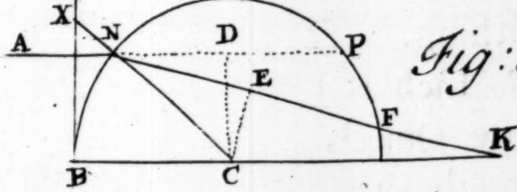


Fig: 62.



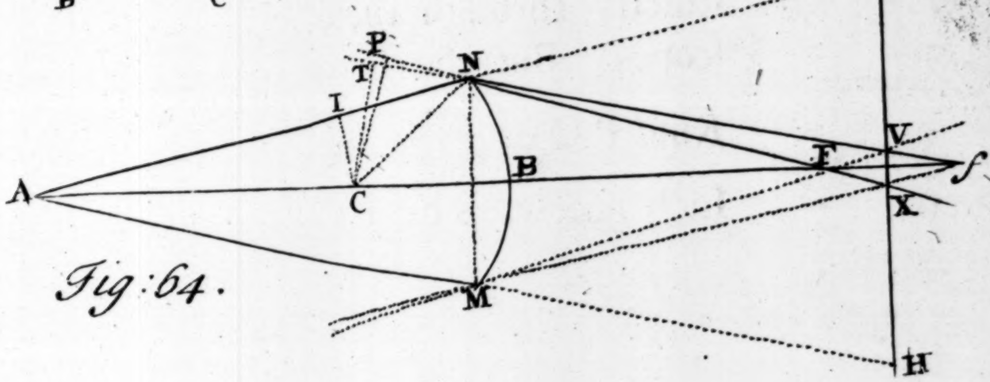


Fig: 64.

come ſenſible, and therefore to be neglected; and the indiſtinct Viſion is only to be attributed to the Errors ariſing from the Heterogeneity of Light. And hence it is manifeſt, that the Perfection of Teleſcopes is not to be ſought from the Conick Sections, but that ſpherical Figures may be equally ſerviceable to this Purpoſe. In Microſcopes indeed the Errors of the homogeneal Rays from the ſpherical Surface of the Object Glaſs on account of the large Aperture do become enormous and very ſenſible. So that thoſe Glaſſes, if they were duly formed into ſome Conick Section, would become a little more perfect. † But I am not ignorant of a Method of correcting thoſe Errors, without the Conick Sections, and by cauſing, that Glaſſes may be made of ſpherical Surfaces, that ſhall refract ſufficiently exact the homogeneal Rays, not to

† I ſuppoſe our Author here means by compoſing the Object-Glaſs of two Glaſſes with Water between them — See the *Scholium* at the End of the firſt *Book* of his *Principia*; and *Prop.* VII. *Part* 1. *Book* 1. of his *Opticks*.

ſay,

ſay, that they ſhall refract far more accurately the oblique Pencils of Rays, than Glaſſes terminated by any other Figures whatſoever. So that I think, ſpherical Surfaces are better accommodated to Dioptrical Uſes than all others.

FINIS.